An analysis of q-TASEP with a random initial condition

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We report on a result for the q-TASEP with a random initial condition. The q-TASEP is a generalization of the standard TASEP (totally asymmetric simple exclusion process) in which on $\mathbb{Z}$ a particle hops to the right nearest neighboring site with rate $1-q^n, 0<q<1$, where n is the gap (the number of empty sites) between the particle and the right nearest particle. The translationally invariant stationary measure is known to be given by the gaps $\eta_i, i \in \mathbb{Z}$ which are independent and each of them distributed as

$$\mathbb{P}[\eta_i = n] = (\gamma;q)_\infty \frac{\gamma^n}{(q;q)_n},\ n=0,1,2,\dots \tag{0.1}$$

for $\gamma \geq 0$. We are interested in the distribution of the integrated current for this stationary measure. For this purpose we consider a modified model, in which hopping rates are particle dependent as $a_i, i \geq 1$ and the left half of the lattice is occupied randomly according to (0.1) with parameter $\gamma = \alpha/a_i$ with $0 < \alpha < a_i$. For the step initial condition (corresponding to $\alpha = 0$), the problem has been studied by using techniques based on the Macdonald process or the duality, but for a random initial condition there was an issue of diverging moments.

We first explain how the q-TASEP can be encoded in a dynamics on a generalized Gelfand-Tsetlin cone and then present a determinant formula for the position of the Nth particle $X_N(t)$,

$$\left\langle \frac{1}{(\zeta q^{x_N(t)+N};q)_\infty} \right\rangle = \det(1-fK)_{L^2(\mathbb{Z})},$$

where $\langle \cdots \rangle$ denotes the expectation, f and K have explicit formulas which allow asymptotic analysis. Two key ingredients for obtaining the formula are (a) Ramanujan's summation formula,

$$\sum_{n\in\mathbb{Z}} \frac{(bq^n;q)_\infty}{(aq^n;q)_\infty} z^n = \frac{(aq;q)_\infty (\frac{q}{az};q)_\infty (q;q)_\infty (\frac{b}{a};q)_\infty}{(a;q)_\infty (\frac{q}{a};q)_\infty (z;q)_\infty (\frac{b}{az};q)_\infty},$$

and (b) the Cauchy determinant formula for the theta function.

If time permits, we would also discuss the application to the stationary q-TASEP. The presentation is based on a collaboration with T. Imamura.