Discrete approximations of continuous determinantal measures: tree representations and tail triviality

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Abstract

A determinantal point process (DPP) μ on S is a probability measure on configuration space S of which correlation functions are given by the determinant of a kernel $\mathsf{K}(x,y)$. If the space is discrete, tail triviality (TT) has been proved by Shirai-Takahashi for Spec(K) $\subset (0,1)$, and Russel Lyons for Spec(K) $\subset [0,1]$. We prove TT of DPPs for continuous spaces.

Let S be a locally compact, complete, and separable metric space equipped with Radon measure m. We consider a (K,m)-determinantal point process μ on S. We assume that K is Hermitian symmetric, of locally trace class, and Spec(K) $\subset [0,1]$, where $\mathsf{K}f(x) = \int_S \mathsf{K}(x,y)f(y)\mathsf{m}(dy)$.

We consider a sequence of partitions $\{\Delta_l\}_{l\in\mathbb{N}}$. Using Δ_l, we define the regular conditional probability $\mu_l = \mu(\cdot|\mathcal{G}_l)$, where

$$\mathcal{G}_l = \sigma[\{\mathsf{s}\in\mathsf{S}; \mathsf{s}(\mathcal{A}) = n\}; \mathcal{A}\in\Delta_l, n\in\mathbb{N}]. \tag{1}$$

If μ_l is TT, then we deduce that μ is TT from the martingale convergence theorem.

To prove TT of μ_l, we introduce a kind of Fourier transform of μ_l and point process ν_l, called tree representations, on a discrete space Ω_l. Ω_l can be regarded as a fiber bundle with tree fiber $\mathbb{I}_l$ and base space Δ_l. The key point is that using the Fourier transform, Perseval's equality and Plancherel's formula, we deduce that ν_l is TT. Moreover, we prove $\mu_l = \nu_l \circ \Pi^{-1}$, here Π is a projection. Hence μ_l inherits TT fromν_l.

This is a joint work with H.Osada.