On scaling limit of a cost in adhoc network model.

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We are interested in giving a mathematical formula of a cost in adhoc network model. In our model, the cost is formulated as an application of first-passage percolation and the motion of devices is random and an asymptotic density of devices is formulated by hydrodynamic limit.

Let $f : \mathbb{Z}_+ \cup \{\infty\} \to [0, \infty]$ be a cost function which is convex. Given a configuration $\eta \in \{0, 1\}^{\mathbb{Z}}$, we make an element $\tilde{x} \in \mathbb{Z}$ corresponds to $x \in \mathbb{Z}$ by $\tilde{x} = \min\{y \in \mathbb{Z}; y \geq x, \eta_y = 1\}$. We define minimized total cost $C(x, y; \eta)$ by

$$C(x, y; \eta) := \inf\{\sum_{i=1}^{n} f(|x_i - x_{i-1}|); x_0 = \tilde{x}, x_n = \tilde{y}, \eta_{x_i} = 1, \text{ for all } 0 < i < n\}.$$

With some extra assumption we can set

$$\gamma_\rho := \inf_l \frac{1}{l} \log E_\rho[\exp C(0, l; \eta)]$$

where E_ρ denotes an expectation with respect to Bernoulli measure on $\{0, 1\}^{\mathbb{Z}}$ with $E_\rho[\eta_0] = \rho$.

Let $\Lambda_N = \{1, 2, \dots, N\}$ be a discrete torus (N is identified with 0) and $X_N := \{0, 1\}^{\Lambda_N}$. Let $\tau_x, x \in \mathbb{Z}$ denote the shift operators. Given a positive local function $c = c(\eta) > 0$ which does not depend on η_0 and η_1 we define a generator of lattice gas L_N by

$$L_N f(\eta) := \sum_{x \in \Lambda_N} \tau_x c(\eta)(f(\eta^{x,x+1}) - f(\eta))$$

where f is a function on X_N and $\eta^{x,y}$ is a configuration defined by $(\eta^{x,y})_x = \eta_y$, $(\eta^{x,y})_y = \eta_x$ and $(\eta^{x,y})_z = \eta_z$ for $z \neq x, y$. Given a probability distribution μ_0^N on X_N we set μ_t^N the law of Markov process generated by $N^2 L_N$ with initial distribution μ_0^N at time t. In this setting we have hydrodynamic limit, i.e., empirical measure converges to $\rho(t, \theta)d\theta$ in probablilty, where $d\theta$ is a Lebesgue measure on $[0, 1)$ and $\rho(t, \theta)$ is give by some classical solution of non-linear diffusion equation. Given a smooth function $\rho : [0, 1) \to [0, 1]$, we set $\Gamma(x, y)$ by

$$\Gamma_{\rho(\cdot)}(x, y) = \min\{\int_x^y \gamma_{\rho(\theta)} d\theta, \int_0^x \gamma_{\rho(\theta)} d\theta + \int_y^1 \gamma_{\rho(\theta)} d\theta\}$$

for $0 \leq x \leq y < 1$.

Theorem 1 Under some assumption on μ_0^N, for all $t \geq 0$ and for all $0 \leq x \leq y < 1$, we have

$$\lim_{N \to \infty} E_{\mu_t^N}[|\frac{1}{N} C(xN, yN; \eta) - \Gamma_{\rho(t,\cdot)}(x, y)|] = 0,$$

where $\rho(t, \cdot)$ is given by hydrodynamic limit.