Sharp interface limite for the stochastic Allen-Cahn equations

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In this talk, we consider the stochastic Allen-Cahn equation with the Diriclet boundary condition;

$$\begin{cases} \dot{u}^\varepsilon(t,x) &= \Delta u^\varepsilon(t,x) + \dfrac{1}{\varepsilon} f(u^\varepsilon(t,x)) + \dot{W}_t^\varepsilon(x), \quad t>0,\ x\in[-1,1], \\ u^\varepsilon(0,x) &= u_0^\varepsilon(x),\ x\in\mathbb{R}, \quad u^\varepsilon(t,\pm 1) = \pm 1,\ t>0, \end{cases}$$

with a small parameter $\varepsilon > 0$. This equation describes a behavior of interface and the parameter ε corresponds to a width of interface. We are interested in the behavior of the solution u^ε when $\varepsilon \to 0$, and we call it the sharp interface limit. In this case, we can expect that the interface motion at the limit is described by a Brownian motion reflected at the boundary of $[-1,1]$. We prove it from the Mosco convergence of Diriclet form which is associated with the $L^2[-1,1]$-valued Markov process $u^\varepsilon(t)$. If we have more time, I would like to present about my previous work of "generation of interface".