SDE representation of infinite particle systems with jumps

Syota Esaki(Kyushu University)
s-esaki@math.kyushu-u.ac.jp

In this talk, we discuss infinite dimensional stochastic differential equation(ISDE) representations of jumps with long range interactions. Our theorems can be applied to the systems with Dyson, Ginibre, Airy and Bessel interactions. Especially, we can give the SDE representations for the these interacting α-stable particle systems for any $\alpha \in (\kappa, 2)$, where κ is the growth order of the density (the 1-correlation function) of μ, that is, $\rho^1(x) = O(|x|^\kappa)$, $|x| \to \infty$.

Suppose that a state space is d-dimensional Euclidian space $\mathbb{R}^d$. Then the configuration space is represented as $\mathfrak{M} = \left\{\xi = \sum_i \delta_{x_i}; \xi(K) < \infty \text{ for all compact sets } K \subset \mathbb{R}^d\right\}$, where δ_a stands for the delta measure at a. We endow $\mathfrak{M}$ with the vague topology. Then $\mathfrak{M}$ is a Polish space. For $x, y \in \mathbb{R}^d$ and $\xi \in \mathfrak{M}$, we write $\xi^{xy} = \xi - \delta_x + \delta_y$ and $\xi \setminus x = \xi - \delta_x$ if $\xi(\{x\}) \geq 1$.

Let μ be a probability measure on $\mathfrak{M}$, which describes an equilibrium measure for the system. We consider a Dirichlet form $\mathfrak{E}$ defined by

$$\mathfrak{E}(f, f) = \frac{1}{2} \int_{\mathfrak{M}} \mu(d\xi) \int_{\mathbb{R}^d} \xi(dx) \int_{\mathbb{R}^d} p(x, y)\{f(\xi^{xy}) - f(\xi)\}^2 dy,$$

with some positive measurable function p on $\mathbb{R}^d \times \mathbb{R}^d$, which is a jump rate of original particles. By using the Dirichlet form we can construct the associated unlabeled particle system [1].

In this talk, we give the ISDE representations for the unlabeled particle systems. We introduce the rate function given by $c(\xi, x; y) = 0$ if $\xi(\{x\}) = 0$, and

$$c(\xi, x; y) = p(x, y)\left(1 + \frac{d\mu_y}{d\mu_x}(\xi \setminus x)\frac{\rho^1(y)}{\rho^1(x)}\right), \quad \text{if } \xi(\{x\}) \geq 1.$$

Here, μ_x is the reduced Palm measure defined by $\mu_x = \mu\left(\cdot - \delta_x \middle| \, \xi(\{x\}) \geq 1\right)$ for $x \in \mathbb{R}^d$, $\rho^1(x)$ is the 1-correlation function of μ and $d\mu_y/d\mu_x$ is the Radon-Nikodym derivative of μ_y with respect to μ_x. Then the labeled process solves the following ISDE:

$$X_j(t) = X_j(0) + \int_{[0,t)\times\mathbb{R}^d\times[0,\infty)} N(dsdudr)u\mathbf{1}\left(0 \leq r \leq c(\Xi(s-), X_j(s-), X_j(s-) + u)\right),$$

for $j \in \mathbb{N}$, where $\Xi(t) = \sum_i \delta_{X_i(t)}$ and $N(dsdudr)$ is a Poisson random point field on $[0, \infty) \times \mathbb{R}^d \times [0, \infty)$ with intensity $dsdudr$. For instance, for the Ginibre interacting α-stable processes with any parameter $\alpha \in (0, 2)$, we see that

$$c(\xi, x; y) = \frac{Z}{|x - y|^{2+\alpha}}\left(1 + \lim_{r\to\infty} \prod_{|x_i|<r} \frac{|y - x_i|^2}{|x - x_i|^2}\right),$$

where Z is a constant and $\xi = \delta_x + \sum_i \delta_{x_i}$. To give the ISDE representation we also use the Dirichlet form technique. This is joint work with Hideki Tanemura (Chiba University).

References

[1] S. Esaki: Infinite particle systems of long range jumps with long range interactions, to appear in Tohoku Mathematical Journal.