Low dimensional topology and number theory XII

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Program
To be announced

Titles and Abstracts

Jesús A. Álvarez López (University of Santiago de Compostela) joint with Yuri Kordyukov and Eric Leichtnam

Deninger’s problem about a trace formula for foliations

This talk is about our progress to understand a problem proposed by Christopher Deninger. He has asked about the validity of certain trace formula for foliated flows using some leafwise reduced cohomology. This was an attempt to establish analogies between Arithmetic Geometry and Foliations.

Let $\mathcal{F}$ be a smooth foliation of codimension one on a closed manifold $M$. Let $\phi^t$ be a flow on $M$, which is foliated in the sense that it maps leaves to leaves. Assume also that the closed orbits and preserved leaves are simple. Then there are finitely many preserved leaves, which are compact and form a compact subset $M^0 \subset M$. Moreover a precise description of the transverse structure of $\mathcal{F}$ can be given.

If $M^0 = \emptyset$, we can consider the usual leafwise reduced cohomology, $\overline{H}(\mathcal{F})$, defined with the leafwise forms that are smooth on $M$. Then the usual action of $\phi^t$ on $\overline{H}(\mathcal{F})$ has a Lefschetz distribution, which satisfies the predicted trace formula. This is a direct consequence of a leafwise Hodge decomposition proved by the first two authors for (transversely) Riemannian foliations.

The problem is much harder when $M^0 \neq \emptyset$, the first difficulty being that the $\overline{H}(\mathcal{F})$ does not seem to be appropriate. Thus we have introduced another version of the leafwise reduced cohomology, $\overline{HI}(\mathcal{F})$, using the complex $I(\mathcal{F})$ distributional leafwise differential forms on $M$ conormal to $M^0$. The forms supported in $M^0$ form a subcomplex $K(\mathcal{F}) \subset I(\mathcal{F})$, with cohomology $HK(\mathcal{F})$. On the other hand, the restriction of the forms in $I(\mathcal{F})$ to $M \setminus M^0$ define a complex $J(\mathcal{F})$, with
reduced cohomology $\overline{H}J(F)$. We have shown that there is a short exact sequence

$$0 \rightarrow HK(F) \rightarrow \overline{HI}(F) \rightarrow \overline{H}J(F) \rightarrow 0.$$  

So the desired Lefschetz distribution of $\phi^t$ on $\overline{HI}(F)$ can be defined as sum of Lefschetz distributions on $HK(F)$ and $\overline{H}J(F)$. Under mild conditions, we have proved that the first distribution has a reasonable definition, which gives the expected contribution of the preserved leaves. The understanding of the second distribution is not complete yet. Besides the expected contribution from the closed orbits, it also involves some kind of distributional periodic eta invariant. We are trying to prove that this undesirable term is well defined and vanishes.

James Borger (Australian National University)
TBA

Hiroshi Goda (Tokyo University of Agriculture and Technology)
Volume formulas using the twisted Alexander invariant and the matrix-weighted zeta function

In this talk, we show some volume formulas of knots using the twisted Alexander invariant. Then, we introduce a digraph obtained from an oriented knot diagram, which is used to study the twisted Alexander polynomial of knots. We show that the inverse of the twisted Alexander polynomial of a knot may be regarded as the matrix-weighted zeta function of the digraph. We will discuss a volume formula using the matrix-weighted zeta function of a graph.

Lars Hesselholt (University of Copenhagen/Nagoya University)
Higher Algebra and Arithmetic

The natural numbers record only the result of counting and not the process of counting. As algebra is rooted in the natural numbers, the higher algebra of Joyal and Lurie is rooted in a more basic notion of number which also records the process of counting. Long advocated by Waldhausen, the arithmetic of these more basic numbers should eliminate denominators. Notable manifestations of this vision include the Bokstedt-Hsiang-Madsen topological cyclic homology, which receives a denominator-free Chern character, and the related Bhatt-Morrow-Scholze integral $p$-adic Hodge theory, which makes it possible to exploit torsion cohomology classes in arithmetic geometry. Moreover, for schemes smooth and proper over a finite field, the analogue of de Rham
Hikaru Hirano (Kyushu University)
TBA

Steven Hurder (University of Illinois at Chicago)
From Number Theory to Cantor dynamics
In this talk, we discuss an application of the dynamical properties of Cantor actions to number theory and some of the questions raised by this connection. A Cantor dynamical system is the action of a countable group $G$ on a Cantor space $X$. The class of equicontinuous Cantor actions is equivalent to the class of arboreal actions. We discuss properties of equicontinuous Cantor actions, and introduce the notions of stable and wild Cantor actions. We then discuss the Cantor actions associated to absolute Galois groups, and give some examples whose associated actions are stable, and those which are wild.

Hiroaki Karuo (RIMS, Kyoto University)
On the reduced Dijkgraaf–Witten invariant of knots in the Bloch group of $\mathbb{F}_p$
For a closed oriented 3-manifold $M$, a discrete group $G$, a 3-cocycle $\alpha$ of $G$, and a representation $\rho: \pi_1(M) \to G$, the Dijkgraaf–Witten invariant is defined to be $\rho^*\alpha[M]$, where $[M]$ is the fundamental class of $M$, and $\rho^*\alpha$ is the pull-back of $\alpha$ by $\rho$. We consider an equivalent invariant $\rho_*[M] \in H_3(G)$, and we also regard it as the Dijkgraaf–Witten invariant. In 2004, Neumann described the hyperbolic volume and the Chern–Simons invariant of $M$ in terms of the image of the Dijkgraaf–Witten invariant for $G = \text{SL}_2\mathbb{C}$ by the Bloch–Wigner map $H_3(M) \to \mathcal{B}(\mathbb{C})$, where $\mathcal{B}(\mathbb{C})$ is the Bloch group of $\mathbb{C}$. Further, in 2013, Hutchinson gave a construction of the Bloch–Wigner map $H_3(\text{SL}_2\mathbb{F}_p) \to \mathcal{B}(\mathbb{F}_p)$ explicitly, where $p$ is prime, and $\mathbb{F}_p$ is the finite field of order $p$.

In this talk, I will show calculations of the reduced Dijkgraaf–Witten invariant of the complements of knots, especially twist knots, where the reduced Dijkgraaf–Witten invariant is the image of the Dijkgraaf–Witten invariant for $\text{SL}_2\mathbb{F}_p$ by the Bloch–Wigner map $H_3(\text{SL}_2\mathbb{F}_p) \to \mathcal{B}(\mathbb{F}_p)$.

Junhyeong Kim (Kyushu University)
Eiko Kin (Osaka University)
Problem on pseudo-Anosov minimal entropies
The aim of this talk is to survey the recent development on the subject. The problem is related to dynamical systems, number theory, and hyperbolic geometry. In the first part of my talk, I will explain the significant role of fibered faces of hyperbolic fibered 3-manifolds for the study of the problem. In the second part, I will talk about other numerical invariants of pseudo-Anosov maps (volume, stable length on the curve complex, for example) and explain the relation to entropies.

Jungin Lee (Pohang University of Science and Technology)
Arithmetic Chern-Simons theory for arithmetic schemes
We introduce the arithmetic Chern-Simons theory for regular flat separated schemes of finite type over rings of integers of number fields. Recently, Geisser and Schmidt generalized duality theorems (Artin-Verdier duality, Poitou-Tate duality and Poitou-Tate exact sequence) to such schemes by using Bloch’s cycle complexes. In this talk, we explain how these duality theorems can be used for a generalization of the arithmetic Chern-Simons theory.

Hiroaki Nakamura (Osaka University)
Geometry and arithmetic of Lissajous geodesics on the equianharmonic modular surface
Starting from the moduli space of Euclidean triangles, we illustrate how choreographic motions of three bodies on Lissajous curves produce a family of closed curves on \( \mathbb{P}^1(\mathbb{C}) - \{0, 1, \infty\} \) and (normalized) geodesics on the upper half plane. We introduce 4-symbol frieze patterns that encode cutting sequences of the geodesics along the Farey tessellation and discuss their relations to odd continued fractions of quadratic surds as well as to a family of Christoffel words. This is a joint work with H.Ogawa and E.Kin.

Yuta Nozaki (Meiji University)
An invariant of 3-manifolds via homology cobordisms
For a closed 3-manifold \( X \), we consider a topological invariant defined as the minimal integer \( g \) such that \( X \) is obtained as the closure of a homology cobordism over a surface of genus \( g \). We prove that the invariant
equals one for every lens space, which is contrast to the fact that some lens spaces do not admit any open book decomposition whose page is a surface of genus one. The proof is based on the Chebotarev density theorem and binary quadratic forms in number theory.

Jeehoon Park (Pohang University of Science and Technology)
TBA

Kenji Sakugawa (RIMS, Kyoto University)
On the regulator formula for Gealy’s zeta elements
In the middle of 1980’s, Beilinson constructed certain elements in higher K-groups of Kuga-Sato varieties, which relate special L-values of elliptic cuspforms. After 20 years, Gealy constructed zeta elements for modular forms in the higher K-groups of Kuga-Sato varieties following ideas of Beilinson and Kato. Then, he tried to apply his construction to the Bloch-Kato’s Tamagawa number conjecture for elliptic modular forms at negative integer points. In this talk, we give a formula of the image of Gealy’s zeta elements under the Beilinson regulator. We also give an application of our formula to the Tamagawa number conjecture.

Ryoto Tange (Kogakuin University)
Non-acyclic SL$_2$-representations of twist knots and (−3)-Dehn surgeries
We study irreducible SL$_2$-representations of twist knots. We first determine all non-acyclic SL$_2$(C)-representations, which turn out to lie on a line denoted as $x = y$ in $\mathbb{R}^2$ and admit a certain common tangent property. Secondly, we prove that the representations on $x = y$ are exactly those which factor through the (−3)-Dehn surgery. Finally, we investigate the $L$-functions of the universal deformations over a CDVR. This is joint work with Anh T. Tran and Jun Ueki.

Zdzislaw Wojtkowiak (Université de Nice - Sophia-Antipolis)
Some remarks about the main conjecture for $\mathbb{Q}$ and the fundamental group of $\mathbb{P}^1(\mathbb{C}) \setminus \{0,1,\infty\}$
Abstract: TBA