Titles and Abstracts

Francis Brown

Title:

The projective line minus 3 points: past, present and future.

Abstract:

After reviewing some of Ihara's seminal work on the fundamental group of the projective line minus 3 points, and describing some of the twists and turns in this story, I will map out where I believe the future of the subject might lie.

Henri Darmon

Title:

The p-adic uniformisation of modular curves by p-arithmetic groups.

Abstract:

In his important work on *Congruence monodromy problems*, Yasutaka Ihara proposed that the group $\Gamma := \mathrm{SL}_2(\mathbb{Z}[1/p])$ acting on the product of a Drinfeld and a Poincaré upper half plane provides a congenial framework for describing the ordinary locus of the j-line in characteristic p. In Ihara's picture, which rests on Deuring's theory of the canonical lift, the ordinary points of the j-line are essentially in bijection with conjugacy classes in Γ that are hyperbolic at p and elliptic at infinity. I will explain how the classes that are elliptic at p and hyperbolic at infinity form the natural domain for a kind of p-adic uniformisation of $X_0(p)$, leading to a conjectural analogues of Heegner points, elliptic units, and singular moduli over ring class fields of real quadratic fields. This is part of a work in progress with Jan Vonk.

Tomoyoshi Ibukiyama

Title:

Ihara lifts and conjectural correspondences between symplectic automorphic forms of genus two.

Abstract:

Around in 1963, a little earlier than Langlands conjecture was announced, Ihara asked a question to generalize the classical Eichler-Shimizu correspondence between automorphic forms of $\mathrm{SL}_2(\mathbb{R})$ and $\mathrm{SU}(2)$ to those of $\mathrm{Sp}(2, \mathbb{R}) \subset \mathrm{GL}(4)$ and its compact twist $\mathrm{USp}(2)$. He also gave a theory of lifting from elliptic modular forms to automorphic

forms of USp(2). This can be regarded as a compact twist version of Saito-Kurokawa lift (1978) and Yoshida lift (1978) but was given much earlier. We give three different conjectural solutions to his problem when discrete subgroups are parahoric locally (two of them were proposed in 1980's but one is new), and also give a precise conjectural description on the images of Ihara lifts.

Yasutaka Ihara

Title:

On $(\infty \times p)$-adic uniformization of curves $\bmod p$ with assigned many rational points:
(I) some reviews; Shimura's and Igusa's works as points of departure;
(II) existence of uniformization is an abelian problem.

Abstract:

"Why prefer to look at global objects at two places, ∞ and p? For local theory, one place, for global theory, all places. . . ; isn't this the proper way?" So have I been often asked, explicitly or implicitly. But mathematics itself invited me to this wonderful view- and observation-spot, a spot for clear focusing and for some small discoveries (e.g. that of towers of curves $\bmod\ p$ having exceptionally many $\mathbb{F}_{p^2}$-rational points). I mean something related to curves $\bmod p$ and generalized Kronecker-Hecke correspondence $T(p)$.

Upon the existence of "$T(p)$", one sees how the τ-uniformization by the Poincaré upper half plane of curves over $\mathbb{C}$ passes over uniquely to a p-adic differential form $\omega = d\tau$, via Schwarzian differential equation. The $\bmod p$ reduced curve X equipped with the set $\mathfrak{S}$ of zeros of $\omega \bmod\ p$ defines:

(i) the tower $\{Y/X\}$ of all etale covers where the points of $\mathfrak{S}$ split completely;

(ii) an abelian $\mathbb{Z}_p^\times$-tower $\{Z_n/X\}$, associated with $\omega_n = \omega \bmod\ p^{n+1}$. It is etale outside $\mathfrak{S}$ but strongly ramified there.

The origin of (i) are towers of Shimura curves of level $\not\equiv 0 \pmod p$, and (ii) are "Igusa tower" associated with the separable part of modular curves of p-power levels $\bmod p$. But I have been interested in these towers from the viewpoint of construction of analogue of $T(p)$ by lifting; from $\bmod p$ to over $\mathbb{Z}_p$. Interested because (a) I was curious to know which curves X are reduction $\bmod\ p$ of Shimura curves, and also (b) the existence of such lifting gives rise to the "arithmetic $(\infty \times p)$-adic fundamental group" which describes all the Frobenius elements in the tower in terms of subgroups and certain type of conjugacy classes. It is an anabelian reciprocity not in the usual Langlands program. Note that Artin L-functions in the case of curves over finite fields are too weak, and cutting extensions by the degree does not fit with the description of reciprocity in our towers.

The existence of the lifting is (as had been pointed out 40 years ago), equivalent to that of the uniformizing differential tower (ii). This is an abelian problem. The lifting to $\bmod p^2$ had been fully studied earlier, in connection with the differential ω_0. This is the Kummer part of the tower. The remaining tower is $\mathbb{Z}_p$-extension, but there are infinity of them over local fields in characteristic p. Thus, to proceed further, we need "rigidification of local parameters using given unique global datum" such as ω_0. In the second part of my talk, I hope to share with you my present interest in this game.

Kohji Matsumoto

Title:

On the theory of M-functions.

Abstract:

M-functions, named by Professor Y. Ihara, are density functions which describe the value-distribution of L-functions. In the case of the Riemann zeta-function, the primitive form of M-functions already appeared in the work of H. Bohr and others in 1930s. In the beginning of the 21st century, Professor Ihara developed a new approach to the theory of M-functions, and obtained a lot of interesting results. In this talk I will survey Ihara's theory, some of which are written jointly with me, mainly on Dirichlet L-functions. I will also report some recent developments in the theory of M-functions, for other type of zeta and L-functions, including some kind of automorphic L-functions.

Arata Minamide

Title:

The Grothendieck-Teichmüller group as an open subgroup of the outer automorphism group of the étale fundamental group of a configuration space.

Abstract:

Let $n \geq 2$ be an integer and k an algebraically closed field of characteristic zero. Write X for the projective line minus $\{0, 1, \infty\}$ over k,

$$X_n \stackrel{\text{def}}{=} \{(x_1, x_2, \dots, x_n) \in X^n | \ \ x_i \neq x_j \ \text{ for } \ i \neq j\},$$

Π_n for the étale fundamental group of X_n, GT for the [profinite] Grothendieck-Teichmüller group, and $\mathfrak{S}_{n+3}$ for the symmetric group on $n+3$ letters. In the 1990's, Ihara showed results indicating that GT is a suitable subgroup of $\mathrm{Out}(\Pi_n)$. In this talk, we will discuss a result to the effect that the natural outer actions of GT and $\mathfrak{S}_{n+3}$ on Π_n determine an isomorphism $\mathrm{GT} \times \mathfrak{S}_{n+3} \xrightarrow{\sim} \mathrm{Out}(\Pi_n)$. This leads to a simple purely group-theoretic characterization of GT. This is joint work with Yuichiro Hoshi and Shinichi Mochizuki.

Masanori Morishita

Title:

Arithmetic topology in Ihara theory — Milnor invariants, Heisenberg covers and triple power residue symbols.

Abstract:

We introduce mod l Milnor invariants of a Galois element associated to Ihara's Galois representation on the pro-l fundamental group of a punctured projective line (l being a prime number), as arithmetic analogues of Milnor invariants of a pure braid. We then show that triple quadratic (resp. cubic) residue symbols of primes in the rational (resp. Eisenstein) number filed are expressed by mod 2 (resp. mod 3) triple Milnor invariants of Frobenius elements. For this, we study certain mod l Heisenberg branched covers of $\mathbb{P}^1$ and the monodromy transformations of certain functions along the pro-l longitudes of Frobenius elements for $l = 2, 3$.

Hiroaki Nakamura

Title:

Arithmetic and combinatorics in Galois fundamental groups.

Abstract:

In his Annals paper in 1986, Y.Ihara introduced the universal power series for Jacobi sums and showed deep arithmetic phenomena arising in Galois actions on profinite fundamental groups. In particular, the explicit formula established by Anderson, Coleman, Ihara-Kaneko-Yukinari opened remarkable connection to theory of cyclotomic fields (Iwasawa theory) and shed new lights on circle of ideas surrounding Grothendieck's philosophy on anabelian geometry as well as various geometric approaches in Inverse Galois theory by many other mathematicians. In this talk, I will illustrate some of these aspects from a viewpoint of Grothendieck-Teichmüller theory.

Iwao Sato

Title:

Ihara zeta function and quantum walk.

Abstract:

After Professor Ihara defined the Ihara zeta function in 1966, the Ihara zeta function was studied in various fields: number theory, algebra, random walk, combinatorics, graph theory, quantum graph, quantum walk, Ising model etc. The Ihara zeta function has four expressions: the Euler product, the exponential generating function, the determinant expression of Hashimoto type, the determinant expression of Ihara type. The determinant expression of Ihara type for the Ihara zeta function discovered by Professor Ihara is a marked one of it, and involves extremely many informations.

In this talk, we state determinant expressions of Ihara type for the Ihara zeta function of a graph and its variations, and then consider the relation between the Ihara zeta

function and quantum walk from viewpoint of their determinant expressions of Ihara type. Recently, it turned out that discrete-time quantum walks on graphs are efficient for the graph isomorphism problem, and various approach are done in the graph isomorphism problem. Emms et al decided spectra for the Grover transition matrix of the Grover walk on a graph, its positive support and the positive support of its square, and so showed that the positive support of the third power of the Grover transition matrix outperforms the graph spectra methods in distinguishing strongly regular graphs. Furthermore, it is found out that the Grover transition matrix is closely related to the edge matrix appeared in the determinant expression of Hashimoto type for the Ihara zeta function of a graph. We determine the characteristic polynomials of them by using the determinant expressions of Ihara type for the Ihara zeta function and the second weighted zeta function of a graph, and directly present spectra for them. Furthermore, we state the structure of the positive support of the nth power of the Grover transition matrix.

If time permits, then we shall state related topics.

Romyar Sharifi

Title:

Modular symbols and arithmetic.

Abstract:

I intend to discuss developments in a program to generalize a conjectured connection between modular symbols in the homology of modular curves reduced modulo an Eisenstein ideal and Steinberg symbols of cyclotomic units in second K-groups of cyclotomic integer rings. The idea that such a connection should exist originated in a study of Professor Ihara's work on the Galois action on an étale fundamental group of the projective line minus three points.

Katsutoshi Yamanoi

Title:

Kobayashi hyperbolicity of the complements of ample divisors in abelian varieties.

Abstract:

In 1996, Siu and Yeung proved that the complements of ample divisors in abelian varieties are Brody hyperbolic. This result was a motivation of my PhD thesis supervised by Professor Ihara. One problem remained open after the work of Siu-Yeung was whether the stronger hyperbolicity, called Kobayashi hyperbolicity, holds for the complements of ample divisors in abelian varieties. In this talk, I will start from the definitions of several concepts of hyperbolicity of complex manifolds, then discuss my recent progress on this problem.