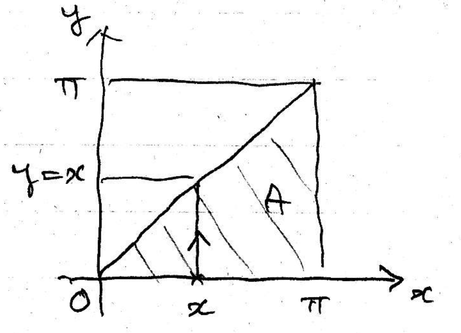
レポート

(1) $\displaystyle\iint_A \cos(x+y)\,dxdy$



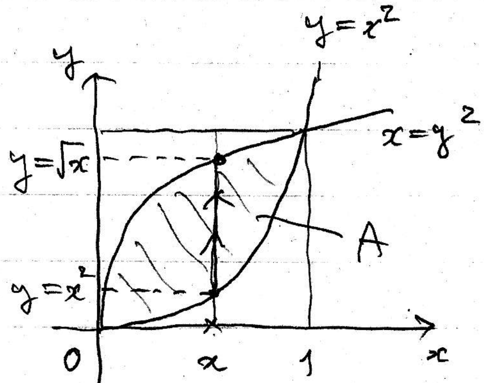
$$= \int_0^{\pi}\left(\int_0^{x} \cos(x+y)\,dy\right)dx$$

$$= \int_0^{\pi}\Big[\sin(x+y)\Big]_{y=0}^{x}\,dx$$

$$= \int_0^{\pi}(\sin 2x - \sin x)\,dx = \left[-\frac{\cos 2x}{2} + \cos x\right]_{x=0}^{\pi}$$

$$= -\frac{1}{2}(\underbrace{\cos 2\pi}_{1} - 1) + (\underbrace{\cos\pi}_{-1} - 1) = -2$$ 」

(3) $\displaystyle\iint_A \sqrt{xy}\,dxdy$



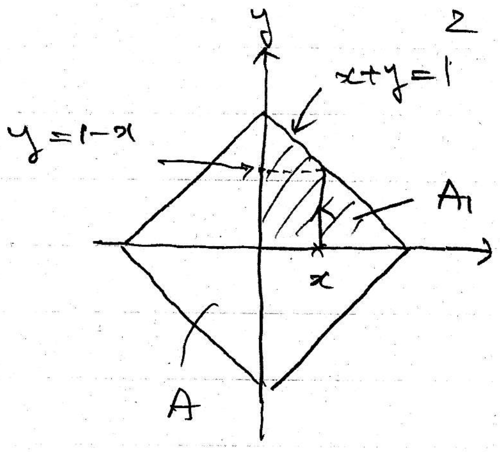
$$= \int_0^{1}\left(\int_{x^2}^{\sqrt{x}} \sqrt{xy}\,dy\right)dx$$

$$= \int_0^{1}\sqrt{x}\left(\int_{x^2}^{\sqrt{x}}\sqrt{y}\,dy\right)dx = \int_0^{1}\sqrt{x}\left[\frac{2}{3}y^{\frac{3}{2}}\right]_{y=x^2}^{\sqrt{x}}dx$$

$$= \int_0^{1}\sqrt{x}\,\frac{2}{3}\left(x^{\frac{3}{4}} - x^{3}\right)dx = \frac{2}{3}\int_0^{1}\left(x^{\frac{5}{4}} - x^{\frac{7}{2}}\right)dx$$

$$= \frac{2}{3}\left[\frac{4}{9}x^{\frac{9}{4}} - \frac{2}{9}x^{\frac{9}{2}}\right]_{x=0}^{1} = \frac{2}{3}\left(\frac{4}{9} - \frac{2}{9}\right) = \frac{4}{27}$$ 」

(5) $\displaystyle\iint_A (|x|+|y|)\,dxdy$



$\displaystyle = 4\iint_{A_1} (|x|+|y|)\,dxdy$

$\displaystyle = 4\iint_{A_1} (x+y)\,dxdy$

$\displaystyle = 4\int_0^1 \left(\int_0^{1-x} (x+y)\,dy\right) dx = 4\int_0^1 \left[xy+\frac{y^2}{2}\right]_{y=0}^{1-x} dx$

$\displaystyle = 4\int_0^1 \left\{x(1-x)+\frac{1}{2}(1-x)^2\right\} dx$

$\displaystyle = 4\int_0^1 \left\{x-x^2+\frac{1}{2}(x-1)^2\right\} dx$

$\displaystyle = 4\left[\frac{x^2}{2}-\frac{x^3}{3}+\frac{1}{6}(x-1)^3\right]_{x=0}^{1}$

$\displaystyle = 4\left(\frac{1}{2}-\frac{1}{3}-\frac{1}{6}(-1)^3\right) = 4\left(\frac{1}{6}+\frac{1}{6}\right) = \frac{4}{3}$ ⌟