A $p$-complete version of the Ganea Conjecture for co-H-spaces

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Abstract. A finite connected CW complex which is a co-H-space is shown to have the homotopy type of a wedge of a bunch of circles and a simply-connected finite complex after almost $p$-completion at a prime $p$.

1. Fundamentals and Results

When $Y$ is a homotopy associative H-space or when $X$ is a (homotopy) associative co-H-space, the set of based homotopy classes $[Z, Y]$ or $[X, Z]$ respectively, is a group natural in the $Z$ argument. If the H-multiplication on $Y$ is not known to be homotopy associative, the induced structure on $[Z, Y]$ is that of an algebraic loop; in particular, left and right inverses exist but they may be distinct. One cannot make as general a statement for $[X, Z]$ when $X$ is a co-H-space. The immediate problem is that whereas the shearing map for an H-space induces isomorphisms of homotopy groups, the co-shearing map for a co-H-space induces isomorphisms of homology groups. This general situation has been well understood for some decades.

We assume that spaces have the homotopy types of CW-complexes, are based and that maps and homotopies preserve base points. A space $X$ is a co-H-space if there is a comultiplication map $\nu : X \to X \vee X$ satisfying $j \circ \nu \simeq \Delta : X \to X \times X$ where $j : X \vee X \hookrightarrow X \times X$ is the inclusion and $\Delta$ the diagonal map. Equivalently, $X$ is a co-H-space if the Lusternik-Schnirelmann category $\text{cat} X$ is one.

Statements (1.1) and (1.2) below were shown to be equivalent in [13], see also Theorem 0.1 of [16].

(1.1) $X$ is a co-H-space and the comultiplication can be chosen so that $[X, Z]$ is an algebraic loop for all $Z$.

(1.2) The space $X$ has the homotopy type of a wedge of a bunch of circles and a simply-connected co-H-space.

Problem 10 in [11] asks “Is any (non-simply-connected) co-H-space of the homotopy type of $S^1 \vee \cdots \vee S^1 \vee Y$ where there may be infinitely many circles and $\pi_1(Y) = 0$?” The positive statement has become known as ‘the Ganea conjecture’.

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1991 Mathematics Subject Classification. Primary 55P45.

Key words and phrases. co-H-space, Ganea’s conjecture, almost completion, atomic space.

This research was partially supported by Grant-in-Aid for Scientific Research (C)08640125 from The Ministry of Science, Sports and Culture.
for co-H-spaces (see section 6 of [1]). The conjecture was resolved thirty years after being raised when the second author constructed in [16] infinitely many finite complexes which are co-H-spaces but which do not have the homotopy type described in Problem 10. This leaves open a p-complete version of Ganea’s conjecture, and probably more difficult, a p-local version (see Conjecture 1.6 of [16]). The rational version was established in [12] as a prime decomposition theorem for an ‘almost rational’ co-H-space. In this note, we address the p-complete problem at a prime p.

Some comments are required on p-completion. The p-completion of a simply connected co-H-space is rarely a co-H-space (unless it is a ‘finite torsion space’ in the sense of [8]) as a wedge of p-complete spaces need not be p-complete; it becomes a co-H-object in a categorical sense, which is adequate for some purposes. More seriously, we are interested in non-simply-connected co-H-spaces and it is shown in [9] that when X is a co-H-space,

$$\pi_1(X)$$ is a free group.

We therefore use fibrewise p-completion which we describe after introducing more notation.

Let X be a co-H-space and $$B = B\pi_1(X)$$, so that B can be chosen as a bunch of circles by (1.3). Let $$i: B \to X$$ represent the generators of $$\pi_1(X)$$ associated with the circles and $$j: X \to B$$ be the classifying map of the universal cover $$\tilde{X} \to X$$. We may assume that $$j \circ i \simeq 1_B$$, and so B is a homotopy retract of X. Also let $$c: X \to C$$ be the cofibre of $$i: B \to X$$, so C is simply connected. One seems tantalizingly close to Ganea’s original conjecture as there are homology equivalences $$X \to B \lor C$$ and $$B \lor C \to X$$ inducing isomorphisms of fundamental groups.

For each prime p, we consider the fibrewise p-completion of $$j: X \to B$$, $$\hat{\iota}_p: X \to \hat{X}_p$$ which commutes with projections to B. The map $$\hat{\iota}_p$$ induces an isomorphism of fundamental groups and acts as standard p-completion on the fibre $$\tilde{X}$$ and so $$\hat{X}_p \simeq \tilde{X}_p$$. Following earlier authors, we refer to this fibrewise p-completion for $$j: X \to B$$ as ‘almost p-completion’. A general reference for fibrewise p-completion is [6], [4] or [18]. Also it is shown in [16] that a co-H-space X is a co-H-space over B up to homotopy, and so $$\hat{X}_p$$ becomes a co-H-object over B in the sense of [17] and [8].

The main result of this note is the following.

**Theorem 1.1.** Let X be a finite, connected, based CW-complex and a co-H-space. After almost p-completion, $$\hat{X}_p$$ has the homotopy type of $$\hat{(B \lor C)}_p$$ where B is a finite bunch of circles and C is a simply-connected finite complex and a co-H-space.

Let $$Y \to B$$ be a fibration with cross-section.

**Corollary 1.2.** The homotopy set $$[\hat{X}_p, \hat{Y}_p]_B$$ inherits an algebraic loop structure from C.

Since C is a simply-connected co-H-space, results of [21] imply that $$\hat{C}_p$$ can be decomposed, uniquely up to homotopy, as a completed wedge sum of simply-connected p-atomic spaces.

**Corollary 1.3.** $$\hat{X}_p$$ has the homotopy type of the almost p-completion of a wedge sum of circles and simply-connected p-atomic spaces.
The general strategy used to prove Theorem 1.1 first occurs in [15] in establishing the Ganea conjecture for complexes of dimension less than 4. The existence of a co-H-multiplication enabled the authors to construct a new splitting \( C \to X \) to obtain a homotopy equivalence \( B \vee C \to X \). In our case, we adopt techniques for simply-connected \( p \)-complete spaces of [14] and [21] for a similar purpose.

The first author expresses his gratitude to the Kyushu University and the second to the University of Aberdeen for their hospitality.

2. Finite co-H-complexes

We give a different proof of Theorem 3.1 of [15] and include a converse statement for completeness.

**Theorem 2.1.** Let \( X \) have the homotopy type of a based CW complex. Then \( X \) has the homotopy type of a finite complex which is a co-H-space if and only if there are a finite bunch of circles, a connected finite complex \( D \) and maps

\[
\rho : B \vee \Sigma D \to X \quad \text{and} \quad \sigma : X \to B \vee \Sigma D
\]

satisfying \( \rho \circ \sigma \simeq 1_X \).

**Proof.** Let \( X \) be a finite complex and a co-H-space. Then by [10], \( X \) is dominated by \( \Sigma \Omega X \), where \( \Sigma \Omega X \simeq \Sigma \pi_1 \vee \Sigma (\vee_{\tau \in \pi} \Omega \tau X) \) and \( \pi = \pi_1(X) \) is a free group. As \( X \) is a finite complex, the rank of \( \pi \) is finite and \( B = B \pi \) is a finite bunch of circles and \( X \) is dominated by \( B \vee \Sigma (\vee_{\tau \in \pi} \Omega \tau X) \). The image of \( X \) can be taken as a finite subcomplex of \( B \vee \Sigma (\vee_{\tau \in \pi} \Omega \tau X) \) and so there is a finite subcomplex \( D \) in \( \vee_{\tau \in \pi} \Omega \tau X \) such that \( B \vee \Sigma D \) dominates \( X \).

Conversely, let \( X \) be dominated by \( B \vee \Sigma W \) where \( B \) is a finite bunch of circles indexed by a finite set \( \Lambda \) and \( W \) is a connected finite complex. Then \( B \vee \Sigma W = \Sigma (\Lambda \vee W) \), and cat \( X \leq \text{cat} (\Sigma (\Lambda \vee W)) = 1 \). Thus \( X \) is a co-H-space. Also \( X \) is dominated by the finite complex \( B \vee \Sigma W \) whose fundamental group is free of finite rank. The finiteness obstruction for \( X \) lies in the Whitehead group \( Wh(\pi) = K_0(\mathbb{Z}[\pi_1(X)])/\pm 1 \) ([20] and [19]) which is zero (see [7] and [2]). Thus \( X \) has a homotopy type of a finite complex. This completes the proof. \( \square \)

Let \( P = \sigma \rho : B \vee \Sigma D \to B \vee \Sigma D \) be the homotopy idempotent given by Theorem 2.1. So \( P \) restricted to \( B \) can be chosen as the inclusion \( \text{in}_B : B \subset B \vee \Sigma D \) and \( P \) restricted to \( \Sigma D \) lifts to \( P_0 : \Sigma D \to B \vee \Sigma D \) where \( B \vee \Sigma D \simeq \vee_{\tau \in \pi} \tau \cdot \Sigma D \) as \( \Sigma D \) is simply connected. As \( \Sigma D \) is a finite complex, \( P_0(\Sigma D) \) is included in a finite subcomplex \( \bigvee_{i=1}^t \tau_i \cdot \Sigma D \). So the restriction of \( P \) to \( \Sigma D \) equals the composition

\[
\Sigma D \xrightarrow{P_0} \bigvee_{i=1}^t \tau_i \cdot \Sigma D \hookrightarrow B \vee \Sigma D \twoheadrightarrow B \vee \Sigma D.
\]

Therefore we have the commutative diagram

\[
\begin{array}{ccc}
B \vee \Sigma D & \xrightarrow{P} & B \vee \Sigma D \\
\downarrow{1_B \vee P_0} & & \downarrow{(\text{in}_B \cdot P)} \\
B \vee \bigvee_{i=1}^t \tau_i \cdot \Sigma D & \xrightarrow{B \vee \bigvee_{i=1}^t \tau_i} & B \vee \bigvee_{\tau \in \pi} \tau \cdot \Sigma D
\end{array}
\]

which plays a crucial role in the next section.
whose homotopy groups are finite p-groups.

\[ \tau \] is a continuous homomorphism of topological groups, where the group structure is chosen so that \( \rho' \circ \sigma' \simeq 1_C \), as \( \rho \circ \sigma \simeq 1_X \). Thus the self map \( P' = \sigma' \circ \rho' \) of \( \Sigma D \) is also a homotopy idempotent. We will investigate the compositions

\[ \hat{\Xi}_p^a \rightarrow \hat{B} \lor \Sigma \hat{D}_p^a \cong \hat{B} \lor \Sigma \hat{D}_p^a \rightarrow \hat{B} \lor \hat{C}_p^a \]

and

\[ \hat{B} \lor \hat{C}_p^a \rightarrow \hat{B} \lor \Sigma \hat{D}_p^a \cong \hat{B} \lor \Sigma \hat{D}_p^a \rightarrow \hat{X}_p^a \]

with an appropriate homotopy equivalence \( \phi \).

3. Almost p-complete co-H-objects

Using the universality of almost p-completion, we have the natural equivalences between homotopy sets.

\[ [(\hat{B} \lor \Sigma \hat{D})_p^a] = [(\hat{B} \lor \Sigma \hat{D})_p^a] = [\Sigma \hat{D}, (\hat{B} \lor \Sigma \hat{D})_p^a] \]

where \( \hat{V} \) denotes the completed wedge sum. Projecting to its factors \( \tau \cdot \hat{\Sigma} \hat{D}_p^a \), we have a map

\[ \beta : [\hat{B} \lor \Sigma \hat{D}]_p^a \rightarrow \hat{\Sigma} \hat{D}_p^a, \hat{\Sigma} \hat{D}_p^a = \prod_{\tau \in \pi} \tau \cdot [\hat{\Sigma} \hat{D}_p^a, \hat{\Sigma} \hat{D}_p^a] \]

to the product and the image of \( \beta \) contains the sum \( \sum_{\tau \in \pi} \tau \cdot [\hat{\Sigma} \hat{D}_p^a, \hat{\Sigma} \hat{D}_p^a] \). Indeed \( \beta \) is a continuous homomorphism of topological groups, where the group structure is inherited from the co-H-space \( \Sigma D \).

We give an alternative description of the closed subgroup which is the image of \( \beta \). Let \( \{ g_\tau \}_{\tau \in \pi} \) denote an element of the product \( \prod_{\tau \in \pi} \tau \cdot [\hat{\Sigma} \hat{D}_p^a, \hat{\Sigma} \hat{D}_p^a] \).

**Proposition 3.1.** The element \( \{ g_\tau \}_{\tau \in \pi} \) lies in the image of \( \beta \) if and only if \( \{ \chi \circ g_\tau \}_{\tau \in \pi} \in \sum_{\tau \in \pi} \tau \cdot [\Sigma D, K] \) for any map \( \chi : \hat{\Sigma} \hat{D}_p^a \rightarrow K \) and any space \( K \) of whose homotopy groups are finite p-groups.

**Proof.** Let \( f : \hat{\Sigma} \hat{D}_p^a \rightarrow \hat{\Sigma} \hat{D}_p^a \) and \( \beta(f) = \{ f_\tau \}_{\tau \in \pi} \in \prod_{\tau \in \pi} \tau \cdot [\hat{\Sigma} \hat{D}_p^a, \hat{\Sigma} \hat{D}_p^a] \).

Since \( (\hat{\Sigma} \hat{D}_p^a) \circ f \) lies in \( \sum_{\tau \in \pi} \tau \cdot [\hat{\Sigma} \hat{D}_p^a, \hat{\Sigma} \hat{D}_p^a] = [\Sigma D, \hat{\Sigma} \hat{D}_p^a] \), the map \( \{ \chi \circ f_\tau \}_{\tau \in \pi} \) lies in \( \sum_{\tau \in \pi} \tau \cdot [\Sigma D, K] \) as required. The converse statement holds by naturality and fundamental properties of p-completion. \( \square \)
Lemma 3.2. \( \beta(\tilde{P}_p^a) \in \sum_{\tau \in \pi} \tau \cdot [\tilde{D}_p, \Sigma \tilde{D}_p]. \)

Proof. The lemma follows from (2.1).

We now recall results from [14] and [21]. We define a homomorphism of near-algebras by mapping homotopy classes of self-maps of \((B\vee \Sigma D)_p\) over \(B\) to the induced endomorphism of \(\tilde{H}_*((B\vee \Sigma D)_p; \mathbb{F}_p) \cong \tilde{H}_*(\Sigma D_p; \mathbb{F}_p)\) \(\approx\) (see [15]) the \(\mathbb{F}_p\)-homology groups of the universal cover

\[
\alpha : [(B\vee \Sigma D)_p, (B\vee \Sigma D)_p]_B \to \text{End}_{\mathbb{F}_p} \{ \tilde{H}_*(\Sigma D_p; \mathbb{F}_p) \}.
\]

When \(B\) is a point, the same definition gives a homomorphism

\[
\alpha_0 : [\tilde{D}_p, \Sigma \tilde{D}_p] \to \text{End}_{\mathbb{F}_p} \{ \tilde{H}_*(\Sigma D_p; \mathbb{F}_p) \}.
\]

The homomorphisms \(\alpha\) and \(\alpha_0\) fit into a commutative diagram

\[
\begin{array}{ccc}
[(B\vee \Sigma D)_p, (B\vee \Sigma D)_p]_B & \xrightarrow{\alpha} & \text{End}_{\mathbb{F}_p} \{ \tilde{H}_*(\Sigma D_p; \mathbb{F}_p) \} \\
\prod_{\tau \in \pi} \tau \cdot [\tilde{D}_p, \Sigma \tilde{D}_p] & \downarrow{\beta} & \\
\sum_{\tau \in \pi} \tau \cdot [\tilde{D}_p, \Sigma \tilde{D}_p] & \xrightarrow{\text{End}_{\mathbb{F}_p} \{ \tilde{H}_*(\Sigma D_p; \mathbb{F}_p) \}} & \sum_{\tau \in \pi} \tau \cdot \text{End}_{\mathbb{F}_p} \{ \tilde{H}_*(\Sigma D_p; \mathbb{F}_p) \}.
\end{array}
\]

The topological radical \(N\) in the compact, Hausdorff space \([\tilde{D}_p, \Sigma \tilde{D}_p]\) is defined by

\[
N = \{ n \in [\tilde{D}_p, \Sigma \tilde{D}_p] \mid h \cdot n \text{ is topologically nilpotent for all } h \}.
\]

The radical \(R\) in the finite ring \(\text{End}_{\mathbb{F}_p} \{ \tilde{H}_*(\Sigma D_p; \mathbb{F}_p) \}\) is defined by

\[
R = \{ r \in \text{End}_{\mathbb{F}_p} \{ \tilde{H}_*(\Sigma D_p; \mathbb{F}_p) \} \mid \text{For any } u, \there exists } v \text{ such that } v(1 + uv) = 1 \}.
\]

Then (see section 3 in [14]), \(\alpha_0\) induces a homomorphism of rings

\[
\alpha'_0 : [\tilde{D}_p, \Sigma \tilde{D}_p]/N \to \left\{ \text{End}_{\mathbb{F}_p} \{ \tilde{H}_*(\Sigma D_p; \mathbb{F}_p) \} \right\}/R
\]

which is a monomorphism onto its image, which can be identified with \(\bigoplus_{i=1}^k M(n_i, \mathbb{F}_q)\) for some \(k\), where \(\mathbb{F}_q\) is a finite field of characteristic \(p\) with \(q\) elements.

Lemma 3.3. There is an isomorphism of rings induced by \(\alpha\).

\[
\alpha' : \sum_{\tau \in \pi} \tau \cdot ([\tilde{D}_p, \Sigma \tilde{D}_p]/N) \to \bigoplus_{i=1}^k M(n_i, \mathbb{F}_q, \pi).
\]

Proof. We identify \(\mathbb{F}_p \pi \otimes \text{End}_{\mathbb{F}_p} \{ \tilde{H}_*(\Sigma D_p; \mathbb{F}_p) \}\) with \(\text{End}_{\mathbb{F}_p} \{ \tilde{H}_*(\Sigma D_p; \mathbb{F}_p) \}\) so that the image \(\mathbb{F}_p \pi \otimes \left( \bigoplus_{i=1}^k M(n_i, \mathbb{F}_q) \right)\) of \(\alpha' = \sum_{\tau \in \pi} \tau \cdot \alpha_0\) becomes \(\bigoplus_{i=1}^k M(n_i, \mathbb{F}_q, \pi)\).
4. The proof of Theorem 1.1

Lemmas 3.2 and 3.3 imply that $\hat{P}_p^a \in [(B \vee \Sigma D)_p^a, (B \vee \Sigma D)_p^a]$ is mapped in homology to a direct sum of idempotents $\bigoplus_{i=1}^k P_i \in \bigoplus_{i=1}^k M(n_i, \mathbb{F}_q, \pi)$. We appeal to work of Bass [3]; each $P_i$ defines an $\mathbb{F}_q, \pi$-homomorphism of $(\mathbb{F}_q, \pi)^{n_i}$ and so there exists an $\mathbb{F}_q, \pi$-isomorphism of $(\mathbb{F}_q, \pi)^{n_i}$, $A_i$ say, such that

$$A_i P_i A_i^{-1} = \begin{bmatrix}
1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 1 
\end{bmatrix} \in M(n_i, \mathbb{F}_q).
$$

This matrix also lies in $M(n_i, \mathbb{F}_q)$. Let a ring homomorphism $\epsilon : \mathbb{F}_q, \pi \to \mathbb{F}_q$ be defined by $\epsilon(\tau) = 1$ and so $\epsilon(P_i)$ represents $\hat{P}_p^a$. We choose $\phi : (B \vee \Sigma D)_p^a \to (B \vee \Sigma D)_p^a$ lying in $\beta^{-1}(\Sigma_{\tau \in \pi} \tau \cdot [\Sigma \hat{D}_p \Sigma \hat{D}_p])$ and a corresponding $\phi' : \Sigma \hat{D}_p \to \Sigma \hat{D}_p$ representing the invertible matrices $\bigoplus_{i=1}^k A_i$ in $M(n_i, \mathbb{F}_q, \pi)$ and $\bigoplus_{i=1}^k \epsilon(A_i)$ in $M(n_i, \mathbb{F}_q)$ respectively. So $\phi$ and $\phi'$ are homotopy equivalences as they induce isomorphisms of homology groups of universal covers by Lemma 3.3. Referring back to (2.2), we have a commutative diagram

$$
\begin{array}{ccc}
\tilde{X}_p^a & \xrightarrow{\bar{a}_p^a} & (B \vee \Sigma D)_p^a \\
\downarrow & & \downarrow \phi \\
\tilde{C}_p & \xrightarrow{\bar{\sigma}_p^a} & \Sigma \hat{D}_p
\end{array}
\quad
\begin{array}{ccc}
(\bar{B} \vee \Sigma D)_p^a & \xrightarrow{\bar{\rho}_p^a} & \tilde{X}_p^a \\
\downarrow & & \downarrow \phi' \\
\tilde{C}_p & \xrightarrow{\bar{\rho}_p^a} & \Sigma \hat{D}_p
\end{array}
$$

The self map $\phi \circ \bar{P}_p^a \circ \phi^{-1}$ of $(B \vee \Sigma D)_p^a$ is a homotopy idempotent represented by the matrix $\bigoplus_{i=1}^k A_i P_i A_i^{-1} = \bigoplus_{i=1}^k \epsilon(A_i) \epsilon(P_i) \epsilon(A_i)^{-1}$ which also represents $\phi' \circ \bar{P}_p^a \circ \phi'^{-1}$. Therefore this matrix also represents $\tilde{s} \tilde{\sigma}_p^a$ where $s = 1_{B \vee (\phi' \circ \tilde{\sigma}_p^a)} : B \vee \tilde{C}_p \to B \vee \Sigma \hat{D}_p$ and $r = 1_{B \vee \Sigma \hat{D}_p} : B \vee \Sigma \hat{D}_p \to B \vee \tilde{C}_p$, and so $\tilde{s} \tilde{\sigma}_p^a \simeq 1_{B \vee \Sigma \hat{D}_p}$. We deduce

$$\beta(\phi \circ \bar{P}_p^a \circ \phi^{-1}) \simeq \beta((s \sigma) \circ p)_p^a \mod \sum_{\tau \in \pi} \tau \cdot N \text{ in } \sum_{\tau \in \pi} \tau \cdot [\Sigma \hat{D}_p \Sigma \hat{D}_p].$$

Let $f = \tilde{r}_p^a \circ \phi \circ \tilde{\sigma}_p^a : (B \vee C)_p^a \to \tilde{X}_p^a$ and $g = \tilde{r}_p^a \circ \phi \circ \tilde{\sigma}_p^a : \tilde{X}_p^a \to (B \vee C)_p^a$. Then

$$g \circ f = (\tilde{r}_p^a \circ \phi \circ \tilde{\sigma}_p^a) \circ (\tilde{r}_p^a \circ \phi \circ \tilde{\sigma}_p^a) = \tilde{r}_p^a \circ (\phi \circ \tilde{r}_p^a \circ \phi \circ \tilde{\sigma}_p^a) \circ (\tilde{r}_p^a \circ \phi \circ \tilde{\sigma}_p^a),$$

whose image by $\beta$ is in $\sum_{\tau \in \pi} \tau - [\Sigma \hat{D}_p \Sigma \hat{D}_p]$ and is homotopic modulo $\sum_{\tau \in \pi} \tau \cdot N$ to that of

$$\tilde{r}_p^a \circ \sigma \circ \tilde{\sigma}_p^a = \tilde{r}_p^a \circ \sigma \circ \tilde{\sigma}_p^a \simeq (1_{B \vee \Sigma \hat{D}_p} \circ (1_{B \vee \Sigma \hat{D}_p} \circ \tilde{\sigma}_p^a).$$

Thus the self map $g \circ f$ of $(B \vee C)_p^a$ over $B$ induces an isomorphisms of homology groups of the universal cover by Lemma 3.3. Therefore
\[ (4.1) \quad g \circ f : (B \vee C)^a_p \to (B \vee C)^a_p \text{ is a homotopy equivalence.} \]

It is routine to check that

\[ (4.2) \quad f \text{ and } g \text{ induce isomorphisms of the } \mathbb{F}_p\text{-homology groups of universal covers,} \]

\[ (4.3) \quad f \text{ and } g \text{ induce isomorphisms of fundamental groups.} \]

We complete the proof by following [5]. Statements (4.1), (4.2) and (4.3) are similar to the conclusion of Lemma 1.6 of [5]. One then makes minor changes to the proof of Theorem 1.5 given there to deduce Theorem 1.1.

References


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