QM $\quad H = -\frac{1}{2}\Delta + V \quad V(x) = |x|^{2m}$

$H\varphi = E\varphi \quad \varphi(x) \sim e^{-|x|^{m+1}}$ 例え $\underline{m=1}$.

$$H e^{-|x|^{\alpha}} = \frac{1}{2}\left(\alpha(\alpha-1)|x|^{\alpha-2} - \alpha^2 |x|^{2\alpha-2} \right) e^{-|x|^{\alpha}} + |x|^{2m} e^{-|x|^{\alpha}}$$

$2\alpha - 2 = 2m \quad \therefore \quad \alpha = m+1.$

- $\| e^{f} g(H) \|$ a estimate

- Carmona estimate

$$(f, e^{tH} g) = \int f(x)\, \mathbb{E}^{x}\left[e^{-\int_0^t V(B_s)ds} g(B_t) \right]$$

$(B_t)_{t\geq 0}$ BM in $(\mathcal{X}, \mathcal{B}, \mathcal{W}^{x})$

- Martingale

$$e^{tE} e^{-\int_0^t V(B_s)ds} \varphi(B_t) = = X_t \quad \text{martingale}$$

... etc

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$$H_p \otimes \mathbb{1} + \mathbb{1} \otimes H_f + g H_i$$ (10分)

$$H_p = -\frac{1}{2}\Delta + V \qquad V = V_+ - V_-$$

$$V_+ \in K_{loc}$$

$$V_- \in L^p(\mathbb{R}^d) \quad p > d/2$$

$$1 \le p < \infty$$

$$V_+(x) = |x|^{2m} \in K_{loc}$$

$$H_f = \int \omega(k) a^+(k) a(k)\, dk \qquad \omega(k) = \sqrt{|k|^2 + m^2}$$

$$H_i = \frac{1}{\sqrt{2}} \int a^+(k) e^{-ikx} \frac{\hat{\varphi}(k)}{\sqrt{\omega(k)}} + a(k) e^{+ikx} \frac{\hat{\varphi}(-k)}{\sqrt{\omega(k)}}\, dk$$

$$\overline{\hat{\varphi}(k)} = \hat{\varphi}(-k). \quad \varphi : \text{real}.$$

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21 ページ以前 $\inf \sigma(H) = E \quad H \overset{\exists}{\varphi} = E\varphi \quad \underline{\underline{\varphi > 0}}$

$$L^2(\mathbb{R}^d) \otimes \mathcal{F} \qquad \mathcal{F} = \bigoplus_{n=0}^{\infty} L^2_{sym}(\mathbb{R}^{dn})$$

$$\cong L^2(\mathbb{R}^d ; \mathcal{F}) \ni F \longmapsto F(x) \in \mathcal{F} \quad \|F\|^2 = \int_{\mathbb{R}^d} \|F(x)\|^2_{\mathcal{F}}\, dx$$

$$(F, e^{-tH} G)_{\mathcal{H}} = \int dx\, \mathbb{E}^x \left[e^{-\int_0^t V(B_s) ds} \left(F(B_0), I_{(0,t)} G(B_t) \right)_{\mathcal{F}} \right]$$

$$I_{(0,t)} = e^{\frac{1}{2} W}\, e^{a^+\left(-\frac{1}{\sqrt{2}} \int_0^t e^{-|s-t|\omega} e^{-ikB_s} ds\right)} e^{-tH_f}$$

$$e^{a\left(-\frac{1}{\sqrt{2}} \int_0^t e^{-|s-t|\omega} e^{ikB_s} ds\right)}$$

$$W = \int_0^t ds \int_0^t ds' \int_{\mathbb{R}^d} \frac{|\hat{\varphi}|^2}{2\omega} e^{-|s-s'|\omega(k)} e^{-ik(B_s - B_{s'})}\, dk.$$

いいろ説明する

$$\| \varphi(x) \|_{\mathcal{F}}^2 = \sum_{n=0}^{\infty} \| \varphi(x)^{(n)} \|_{\mathcal{F}_n}^2 \geq \| \varphi(x)^{(0)} \|_{\mathcal{F}_0}^2$$

$$= (\mathbb{1}, \varphi(x))_{\mathcal{F}}^2 \qquad \mathbb{1} \text{ Fock vacuum}$$

$$(\Psi, \varphi(x))_{\mathcal{F}} \underset{a.e.}{=} \mathbb{E}^x \left[e^{-\int_0^t V(B_s) ds} (\Psi, I_{(0,t)} \varphi(B_t))_{\mathcal{F}} \right]$$

$$(\mathbb{1}, \varphi(x))_{\mathcal{F}} = \mathbb{E}^x \left[e^{-\int_0^t V(B_s) ds} e^W (\mathbb{1}, \varphi(B_t))_{\mathcal{F}} \right]$$

$$\overset{\shortparallel}{\rho_{\mathbb{1}}}(x) = \mathbb{E}^x \left[e^{-\int_0^t V(B_s) ds} e^W \rho_{\mathbb{1}}(B_t) \right] = \rho_\Psi(x)$$

<u>Lem</u> (1) $\rho_\Psi(\cdot)$ cont

(2) $\rho_\Psi(x) > 0$.

<u>Cor</u> $\inf_{x \in K} \rho_\Psi(x) = c \underset{\exists}{>} 0$

$\begin{pmatrix} K \subset \mathbb{R}^d \\ \text{cpt} \end{pmatrix}$

$$\rho_{\mathbb{1}}(x) \geq \mathbb{E} \left[e^{-\int_0^t V_+(B_s + x) ds} \underbrace{e^W (\mathbb{1}, \varphi(B_t + x))} \right]$$

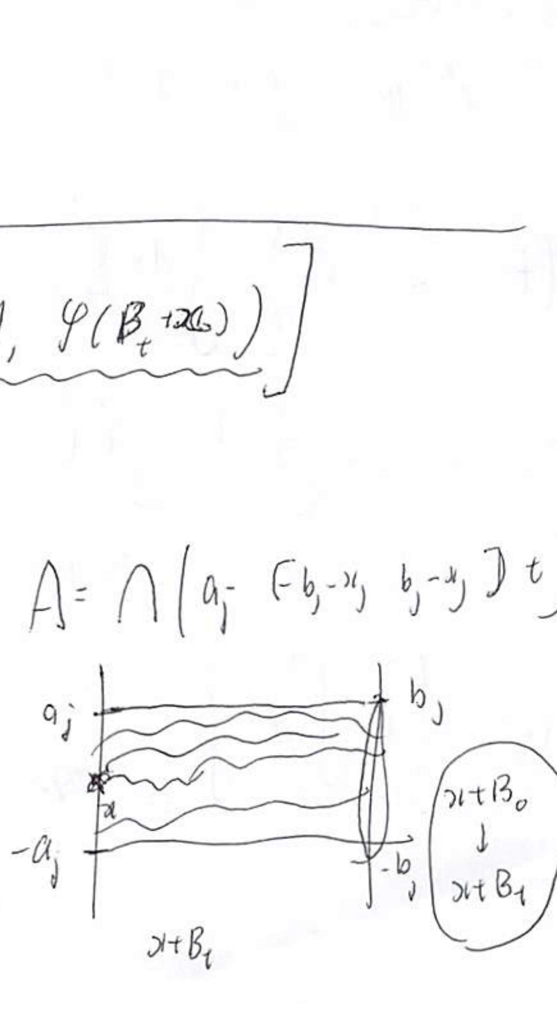
$a_1 \cdots a_d$, $b_1 \cdots b_d$, $\alpha_1 \cdots \alpha_d$

$[-a_j, a_j] \cap [-x_j - b_j, -x_j + b_j] > 2\alpha_j$

$\underline{\alpha_j^2 / t > \beta}$ $\underline{a_j/2 > \alpha_j}$

$\exists$

$A = \cap \left(a_j \quad [-b_j - x_j, b_j - x_j] \, t \right)$



$$\left(\mathbb{1},\ \varphi(B_t+x)\right) \geq \overset{\exists}{\varepsilon}(b)$$

$$e^{W}\left(\mathbb{1},\ \varphi(B_t+x)\right) \geq \varepsilon(b)\, e^{-\sqrt{t}\,\|\hat{\varphi}/w\|_m/\varepsilon(b)}$$

$$e^{-\int_0^t V_+(x+B_s)\,ds} \geq e^{-\int_0^t W_a(x)\,ds} = e^{-t W_a(x)} \quad \text{on } A$$

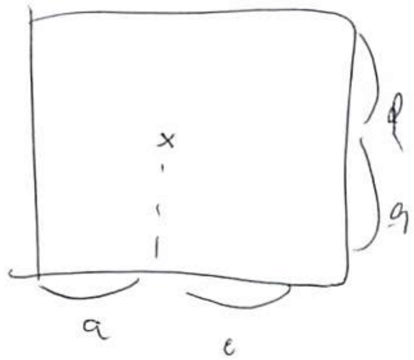
$$\geq e^{tE}\, e^{-t W_a(x)}\, \varepsilon(b)\, e^{-\sqrt{t}\,\|\hat{\varphi}/w\|_m/\varepsilon(b)}\, \underline{\underline{W(A)}}$$

$$\geq \prod_{j=1}^{d} \frac{\alpha_j}{\sqrt{2\pi t}}\, F\left(\frac{a_j^2}{t}\right) e^{-\frac{a_j^2}{2t}}$$

~~F(~~) F は関数

$V = V_+ - V_-$ $\quad \underline{V_+ \leq r|x|^{2m}}$ $\quad x \in K^c$ cpt

$t = |x|^{-(m-1)}$ $\quad a_j = 1+|x_j|$ $\quad \alpha_j = 1/2$ $\quad b = 1.$



$$W_a(x) = \sup_y V(y)$$