Quantum Rabi's model and non-commutative harmonic oscillators

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September 14, 2014

Abstract

The quantum Rabi model is known to be the simplest model used in quantum optics to describe interaction of light and matter beyond the harmonic oscillator (see e.g. [3, 8]). Although this model has had an impressive impact on many fields of physics, only recently in 2011 could this model be declared solved by D. Braak [1] (see also [2]). Introduced over 70 years ago, its applications range from quantum optics, magnetic resonance to solid state and molecular physics.

The non-commutative harmonics oscillator (NcHO) is a self-adjoint, parity-preserving ordinary differential operator of order two with non-commutative coefficient. The NcHO was introduced purely in mathematics context in 1999 [7] (see [6] for the recent progress). Although its spectrum is not still very clear, the deep number theoretic properties of the spectral zeta function shows the family of NcHOs has rich mathematical structure and that's why finding out an inter connection via representation theory between NcHOs and Rabi -kind models would be interesting.

In this talk, a non-trivial relation between the quantum Rabi model and the NcHO discovered recently from the representation theoretic viewpoint in terms of their Heun ODE pictures [10] will be given. In relation with this, certain representation theoretic interpretation of the exceptional eigenstates [4, 5] of the quantum Rabi model will be discussed [11]. Further, if time allows, some number theoretical results, which may relate the Rabi model, will also be discussed (see [9] and references therein).

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