H self-adjoint op $\mathcal{H}$, $D \subset \mathcal{H}$

$[H, {}^{\exists}T] = -i$ on D $\quad$ T - time op

$e^{-itH} e^{isT} = e^{its} e^{-isT} e^{-itH}$ $\quad$ (H,T) WR

WR $\Rightarrow$ CCR $\quad$ $(H,T) \cong \oplus (P,Q)$

$T e^{-itH} = e^{-itT}(T+t)$ $\quad$ (H,T) wwR

WR $\Rightarrow$ wwR $\Rightarrow$ CCR $\quad$ (Miyamoto 2001)

wwR $\Rightarrow \begin{cases} H > -\infty \Rightarrow T \text{ has no s.a. ext} \\ \sigma(H) = \sigma_{ac}(H) \end{cases}$

Ex 1 $\quad (H,T)$ wwR $\overset{\text{strong}}{\Rightarrow} (f(H), {}^{\exists}T_f)$ wwR

$f \in C^2(\mathbb{R}\setminus K)$, $\quad L = \{\lambda \in \mathbb{R}\setminus K ;\ f'(\lambda) \neq 0\}$

$|K|, |L| = 0$ $\quad T_f = \overline{\frac{1}{2}\left(T f'(H)^{-1} + f'(H)^{-1} T\right)|_D}$

Ex 2. $\quad (\Delta, T_{AB})$ wwR

$T_{AB} = \overline{\sum \frac{1}{2}\left(Q_j P_j^{-1} + P_j^{-1} Q_j\right)}_{D_j}$

Ex 3. $\quad H = -\frac{1}{2}\Delta + V$, $\quad H_{ac}$ $\quad {}^{\exists}\operatorname*{s-lim}_{t \to +\infty} e^{itH} e^{-itH_0} = \Omega$

$(= H_0 + V)$

$(H_{ac}, \Omega T_{AB} \Omega^*)$ wwR $\quad / \quad e^{-itH}\Omega = \Omega e^{-itH_0}$

280000
76800
44520
5万円 4

$$\sigma(H) = \sigma_{disc}(H) = \{E_j\}_{j=1}^{\infty} \qquad E < E_2 <$$

<u>Ass</u> $\sum \frac{1}{E_j^2} < \infty$ 　　有限次元, 有界

$H e_n^{\alpha} = E_n e_n^{\alpha} \quad \alpha = 1, \cdots M_n$

$\bar{e}_n = \frac{1}{\sqrt{M_n}} \sum e_n^{\alpha} \qquad \mathcal{E}\text{-span}\{\bar{e}_n - e_m\}$

$\Rightarrow \exists T \ \text{st} \ [H, T] = -i$ on $\mathcal{E}$ 　dense である

weak time op

<u>Thm</u> (1) $\sigma_{sc}(H) = \phi$ 　(2) $\exists T_{ac}$ ~~on $\mathcal{H}_{ac}$~~ strong time for H_{ac}

(3) $\sigma(H_p) = \{0\} \cup \{E_j\}_{j=1}^{\infty} \quad \sum E_j^2 < \infty$

$E < E_2 < 0$

GWT

$\exists T_H \ \mathcal{H} \times \mathcal{H} \to \mathbb{C}$ quadratic Form s.t

$T_H(H\phi, \psi) - T_H(\phi, H\psi) = -i(\phi, \psi)$

$\forall \phi, \psi \in D(T_{ac}) \oplus H_p^{-1}\mathcal{E}$ 　　$\mathcal{E}\text{-span}\{\bar{e}_n \ e_m\}$

$\frac{1}{E_j}$ 　$\frac{1}{H_p} \sim T_d \qquad \left(\frac{1}{H_p}\right)^{-1} \sim T_d'$, 　<u>$T_{ac} \oplus T_d$</u>

<u>Cor</u> $f \in C^2(\mathbb{R} \setminus K)$, $L = \{\lambda \in \mathbb{R} \setminus K ; f'(\lambda) = 0\}$

$\sum f(E_j)^2 < \infty$ 　　$|K|, |L| - 0$

$\Rightarrow \exists T_{f(H)} \ \mathcal{H} \to \mathcal{H}$

GWT for $f(H)$

$-\frac{1}{2}\left[(T_d \varphi, H_p^{-2}\psi) + (H_p^{-2}\varphi, T_d)\right]$

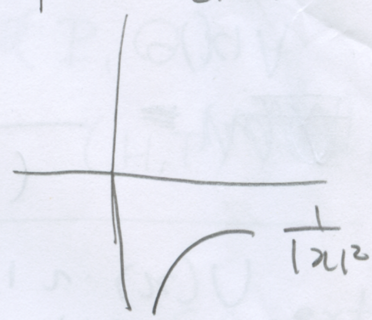
Ex $H = -\frac{1}{2}\Delta + V$ $d=3$

$\sigma_{sc}(H) = \phi$, $\exists$ wave op,   # negative e.v $= \infty$

$\{$positive ev$\} = \phi$ $\sum E_j^2 < \infty$

Agmon   Lieb-Thirring

$V(x) = \dfrac{U(x)}{(1+|x|^2)^{1/2+\varepsilon}}$



$U<0$ cont spherically sym.

$U(x) \sim \dfrac{1}{|x|^\alpha}$ α, $2\varepsilon+\alpha<1$

Ex $H = -\frac{1}{2}\Delta \quad \frac{1}{|x|}$

Ex (H, T) GWT

$\Updownarrow$ $e^{-\beta H}$ に対する $\exists$ GWT T

∘ $\psi(H)$?

∘ $\#\sigma_p(H) < \infty$?