On deriving classical potentials from interactions of particles and quantized fields

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The aim of this talk is to consider a scaling limit of a Hamiltonian describing an interaction of nonrelativistic particles and a quantized field. Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be two Hilbert spaces over $\mathbb{C}$. We consider a self-adjoint operator $\mathbf{H}_\beta(\Lambda)$ with a scaling parameter $\Lambda > 0$ acting in the tensor product of $\mathcal{H}_1$ and $\mathcal{H}_2$, $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$, which is of the form

$$\mathbf{H}_\beta(\Lambda) = H_1 \otimes I + \Lambda\alpha H_{int} + \Lambda^\beta I \otimes H_2, \quad \beta \geq 1.$$

Here H_1 and H_2 are self-adjoint operators in $\mathcal{H}_1$ and $\mathcal{H}_2$, respectively, H_{int} is a symmetric operator in $\mathcal{H}$ and $\alpha \in \mathbf{R}$ is a coupling constant. Introducing a renormalization $E_\beta(\Lambda)$ which goes to infinity or minus infinity as $\Lambda \to \infty$ in some sense, we want to investigate the strong limit of $\exp\left(-it\left(\mathbf{H}_\beta(\Lambda) - E_\beta(\Lambda)\right)\right), t \in \mathbf{R}$ as $\Lambda \to \infty$. In this talk, we shall limit ourselves to discuss the "generalized spin-boson model (GSB)", which is defined as an operator acting in $\mathcal{H}_1 \otimes \mathcal{F}$, where $\mathcal{F}$ denotes a Boson Fock space, by

$$\mathbf{H}_{GSB} = H_1 \otimes I + \alpha \sum_{j=1}^{N} B_j \otimes \phi(\lambda_j) + I \otimes H_b.$$

Here B_j are symmetric operators in $\mathcal{H}_1$, $\phi(\lambda_j)$ and H_b are field operators with momentum cut-off functions λ_j, and the free Hamiltonian in $\mathcal{F}$, respectively.