Abstracts

INTERNATIONAL WORKSHOP

Avoided? crossing of eigenvalue curves

M. Wakayama (Kyushu) The *non-commutative harmonic oscillator* (NcHO) in a normal form is given by

$$Q = Q_{\alpha,\beta} = \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \Big\{ -\frac{1}{2}\frac{d^2}{dx^2} + \frac{1}{2}x^2 \Big\} + \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \Big\{ x\frac{d}{dx} + \frac{1}{2} \Big\}.$$

One knows that Q has only a (positive) discrete spectrum under the condition $\alpha, \beta > 0$, $\alpha\beta > 1$. Still, nothing about the real/ practical shape of eigenvalues is known when $\alpha \neq \beta$ (though one observes numerically a phenomenon of "avoided ? crossings of eigenvalue curves" when these parameters are changing). Thus, in place of investigating the spectrum directly, we look at what properties the spectral zeta function $\zeta_Q(s)$ of the NcHO may have. In the talk, special emphasis will be put on a description of special values $\zeta_Q(k)\,(k = 2, 3, 4, ...)$. Actually, on the study of such special values, certain elliptic curves, modular forms, (newly defined) differential Eisenstein series and Eichler's cohomology groups come up naturally. I will try to touch upon an expecting strategy on feedback from this number theoretical study to the spectrum of NcHO.

S. Taniguchi (Kyushu) Among stochastic approaches to analysis, a key role is played by the expectation representation of heat semigroups via the Brownian motion on manifold. In particular, combined with the Malliavin calculus, the representation gives a rigorous explanation of Feynman path integral type understanding of heat kernels. The representation is available not only for heat equations for scalar functions but also those for differential forms. In the talk, a short review on the expectation representation will be given, and then an extension of the idea to the NCHO will be presented.

Y. Watanabe (Kyushu) This talk presents a computer-assisted proof of solutions of the Orr-Sommerfeld equation describing hydrodynamic stability of Poiseuille flow. Numerical verification methods for computing eigenpair enclosures and eigenvalue exclosures for this non-selfadjoint eigenvalue problem are described. Some verification results confirm the effectiveness of the method.

A. Arai (Hokkaido) We review some aspects of relations between quantum fields (Fock space theories) and number theory. It is shown that some well-known formulae for arithmetical functions (the Riemann zeta function, the Liouville function, Dirichlet series, Möbius function, ...) can be derived from applications of statistical mechanics of quantum fields on Fock spaces.

A. Arai, Infinite-dimensional analysis and analytic number theory, Acta Appl. Math. 63 (2000), 4178.

J. Park (KIAS) The Bergman Tau function is a holomorphic function defined over Teichmüller spaces. This satisfies modular property with repsect to the mapping class group. In this talk, we will explain an infinite product expression of the Bergman Tau function. This can be considered as a generalization of the Dedekind eta function to higher genus case. The complex valued Chern-Simons functional will be introduced for this infinite product expression. We will also explain some corollaries of this result about the eta invariant and a Polyakov type formula.

H. Ochiai (Kyushu) In this talk, I will talk about the relation between the spectral problem of the non-commutative harmonic oscillator of size two in the positive elliptic case and the second order ordinary differential equation on the complex projective line with an accessory parameter, so called Heun's equation. I also mention the multiplicity of the discrete spectrum of the non-commutative harmonic oscillator in the 'odd sector'.

K. Takemura (Chuo) Heun's differential equation is a canonical form of second-order ordinary differetial equations with four singularities which are all regular. A canonical form in the case of three singularities is the hypergeometric differential equation of Gauss, and Heun's equation is a generalization of it. After explaining basic properties of Heun's equation, we introduce some special solutions of Heun's differential equation, i.e. polynomial-type solutions and solutions in the case that one of the regular singularities is apparent. We show that these solutions are related by Euler's integral transformation.

M. Hirokawa (Okayama) The recent cutting-edge technology in circuit QED convinces us of the division between regimes of strong and ultra-strong couplings. There has not been a theory exactly to explain the transition and strictly to determine the value of the coupling strength of the division. In my talk I would like to introduce a method of mathematical modeling to explain the transition by using two notions, the chirality quantum phase transition and the ground state phase transition.

T. Yamamoto (NEC) I will introduce our experimental activities on the superconducting qubits, focusing on the avoided level crossings observed in qubit-qubit and qubit-resonator coupled system. They include the demonstration of quantum algorithm and the dispersive readout.

K. Semba (NTT) We report evidence of coherent strong coupling between dissimilar macroscopic quantum systems i.e., a superconducting macroscopic artificial atom (a flux qubit) and a macroscopic number of electron spins in the form of nitrogenvacancy color centres in diamond. We observed clear vacuum Rabi splitting in the energy spectrum of the coupled system and also observed vacuum Rabi oscillations.

V. Zagrebnov (Marseille) We propose a mathematical model of a micro-wave cavity interacting with atomic beam. For a beam of randomly excited atoms this repeated interaction produces a non-stationary unlimited pumping of the perfect cavity by photons. Additional dissipative Kossakowski-Lindblad dynamics leads to a non-equilibrium stationary steady state.

K. Kimoto (Ryukyus) The non-commutative harmonic oscillator (NCHO) is a non-commutative and higher-dimensional version of the well-known quantum harmonic oscillator. In contrast to the quantum harmonic oscillator, accurate analysis of the spectrum of the NCHO still remains out of reach. Nevertheless, its spectral zeta function can be analyzed rather nicely. In particular, we can obtain various arithmetically interesting properties from the special values of the spectral zeta function, which may reflect the "richness" of the spectral structure of the NCHO as well as give us some expectation on a capability to analyze the spectrum itself. In the talk, we will show the results on these special values as well as derivative arithmetics.

T. Ichinose (Kanazawa) In this lecture we want to talk about an extension of the improved Sobolev embedding theorem for single-valued functions to the case of vector-valued functions. It is involved with the three-dimensional massless Dirac operator together with the three- or two-dimensional Weyl–Dirac (or Pauli) operator, the Cauchy–Riemann operator and also the four-dimensional Euclidian Dirac operator. This is joint work with Yoshimi Saito, University of Alamaba at Birmingham.

K. Schmidt (Vienna) If G is a discrete amenable group, every element f in the integral group ring ZG defines a measure-preserving action α_f of G by automorphisms of a compact abelian group X_f. The entropy of this 'algebraic' action α_f is an interesting quantity. If $G = Z^d$, it is the logarithmic Mahler measure of f. If G is residually finite and the action α_f is expansive, it is the logarithmic growth rate of the number of periodic points of α_f, which in turn coincides with the logarithm of the Fuglede-Kadison determinant associated with f (viewed as an element of the group von Neumann algebra of G). It was recently shown by Hanfeng Li and Andreas Thom that the entropy of α_f coincides with the logarithm of the Fuglede-Kadison determinant associated with f whenever G is discrete and amenable. An intriguing aspect of the connection between Mahler measure and entropy is that the shift-actions on some Z^d-lattice systems (like sandpiles and certain dimer models) also have entropies given by Mahler measures. This coincidence of the entropies of such systems with those of algebraic systems is mysterious.

This talk is largely based on joint results with Deninger, Lind and Verbitskiy.

N. Kurokawa (TITech) We construct a new kind of zeta functions for schemes and study their analytic nature and special values. Special values at negative integers are related to Euler characteristics via "Euler's identity".