RIMS2015 10/6 (火) ネット

$$H = H_p \otimes 1 + 1 \otimes H_f + g H_I$$

$$H_p = -\frac{1}{2}\Delta_x + V, \quad H_f = \int \omega a^*(k) a(k)\, dk \qquad H_I = \frac{1}{\sqrt{2}} \int a^*(k) e^{-ikx} \frac{\hat{\varphi}}{\sqrt{\omega}} dk + h.c$$

$\hat{\varphi}(k) = e^{\varepsilon |k|^2/2} 1_{|k|>\lambda}$ 特別な f_t をとる。

$$E_{Ren} = -\frac{g^2}{2} \int \frac{|\hat{\varphi}|^2}{\omega} \frac{1}{\omega + |k|^2/2} dk \qquad \not\ni \frac{1}{\omega + |k|^2/2}$$

Nelson 1964, GHL 2014,

$$s\text{-}\lim_{\varepsilon \downarrow 0} e^{-T(H - g^2 E_{Ren})} = \exists\, e^{-TH_\infty}$$

E_{Ren} は何か?

形式的な計算 $V=0$ $E(g^2) = \inf \sigma(H)$

$$H = \int^\oplus H(p)\, dp \qquad H(p) = \frac{1}{2}(p - P_f)^2 + H_f + g^2 H_I(0)$$

where $P_f = \int k\, a^*(k) a(k)$, $\quad H_I(0) = \frac{1}{\sqrt{2}} \int a^*(k) \frac{\hat{\varphi}}{\sqrt{\omega}} dk + h.c$

$$E_p(g^2) = \inf \sigma(H(p))$$

Prop $E_0(g^2) = E(g^2)$

$H(0)\varphi = E_0(g^2)\varphi$

$H\varphi = E\varphi$ $\quad g=0$

$H'\varphi + H\varphi' = E'\varphi + \cancel{E\varphi'}$

$H''\varphi + 2H'\varphi' + H\varphi'' - E''\varphi + 2E'\varphi' + E\varphi''$

$\Downarrow$

$H_I(0)\Omega + H_0\varphi' = E'\Omega \to E = 0$

$2H_I(0)\varphi' + H_0\varphi'' = E''\Omega$

$E'' = -2(\Omega, H_I(0) H_0^{-1} H_I(0)\varphi)$ になる

$E_{Ren} \quad E_{Ren} - \frac{1}{2}E''$

$g^2 E_{Ren}$

$$E(g^2) = 0 + g^2 E_{Ren} + g^4 a_2 + \cdots$$

と思われる

Conjecture

$$\lim_{g \downarrow 0} \left| \frac{E(g^2) - g^2 E_{Ren}}{g^2} \right| = 0$$

Gubinelli – H– Lőrinczi

$$S_\varepsilon = \int_{-T}^{T}\int_{-T}^{T} W(B_t - B_s, t-s)$$

$$(\mathbb{1}, e^{-2TH(0)}\mathbb{1}) = \mathbb{E}\left[e^{\frac{g^2}{2}S_\varepsilon}\right]$$

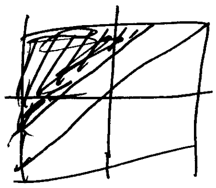
の対角成分 $\times \frac{g^2}{2} = E_{ren}$

<u>Lem</u> $$\left|\frac{E_0(g^2) - g^2 E_{ren}}{g^2}\right| \le g^2 b + \frac{1}{2}C(\tau) \qquad (\tau>0)$$

$$C(\tau) = 8\pi\int_\lambda^\infty e^{-r\tau}\,dr$$

∵ $S_\varepsilon = S_\varepsilon^{OD} + Y_\varepsilon + Z_\varepsilon + 4T\rho_\varepsilon(0,0)$, where

$$\rho_\varepsilon(x,t) = \int \frac{|\hat\varphi|^2}{2\omega}\,\boxed{e^{-|t|\omega}\, e^{-ikx}}\,\frac{1}{k^2/2+\omega}\,dk$$



$$S^{OD}_\varepsilon = 2\int_{-T}^{T} ds \int_{[s+\tau]}^{T} dt\, W$$

$$Z_\varepsilon = -2\int_{-T}^{T}\rho_\varepsilon(B_{[s+\tau]} - B_s, [s+\tau]-s)\,ds$$

$$Y_\varepsilon = 2\int_{-T}^{T}ds\int_s^{[s+\tau]}\nabla\rho_\varepsilon(B_t-B_s, t-s)\,dB_t - \int_{-T}^{T}\Phi_t\,dB_t,$$ when

$$\Phi_t = 2\int_{t-\tau}^{t}\nabla\rho_\varepsilon(\cdot)\,ds$$

$$\rho_\varepsilon(B_t,t)-\rho_\varepsilon(0,0) = \boxed{\int\nabla\rho_\varepsilon\,dB_t + \frac{1}{2}\Delta\rho_\varepsilon\,dt + \int\partial_t\rho_\varepsilon\,dt}$$

$\Downarrow$

$$+4T\rho_\varepsilon(0,0) - \int_0^T W\,dt$$

$$|Z_\varepsilon| \le 2M + (\log 2T)$$

$$|S^{OD}_\varepsilon| \le C(\tau)\,T$$

$$\mathbb{E}\left[e^{\frac{g^2}{2}S_\varepsilon}\right] \le \mathbb{E}\left[e^{\frac{g^2}{2}\left(2M+\log(2T)-1+C(\tau)T+Y_\varepsilon\right)}\right]$$

$$\left|E(g^2)+g^2\rho_\varepsilon(0,0)\right| \le \lim_{T\to\infty}\frac{1}{T}\log e^{\frac{g^2}{2}\left(2M+\log 2T - 1 + C(\tau)T\right)}\,\mathbb{E}\left[e^{\frac{g^2}{2}Y_\varepsilon}\right]$$

$$= \frac{g^2}{2}C(\tau) + \lim_{T\to\infty}\frac{1}{T}\mathbb{E}\left[e^{Y_\varepsilon}\right]$$

$$\frac{g^2}{2}\int_T^T \Phi_t \, dB_t - \frac{g^4}{4}\int |\Phi_t|^2 + \frac{g^4}{4}\int |\Phi_t|^2$$

↙ martingale

$$\mathbb{E}\left[e^{\frac{g^2}{2}Y_\varepsilon - \frac{g^4}{4}\int \Phi^2 + \frac{g^2}{4}\int \Phi^2} \right] \le \mathbb{E}\left[e^{g^2 Y_\varepsilon - \frac{g^4}{2}\int \Phi^2} \right]^{1/2} \mathbb{E}\left[e^{\frac{g^2}{4}\int \Phi^2} \right]^{1/2}$$

$$\le \mathbb{E}\left[e^{\frac{g^4}{4}\int |\Phi|^2} \right] \le e^{g^4 T b}$$

<u>Cor 1</u> $\lim_{g \to 0} \left| \frac{E(g^2) - E^{g^2}_{ren}}{g^2} \right| = 0$

∵ $\lim_{g \to 0} \left| \frac{E(g^2) - E^{g^2}_{ren}}{g^2} \right| \le \frac{1}{2} C(T)$ $\quad C(T) \to 0 \quad (T \to \infty)$

<u>Cor 2</u> $E(g) - E^{g^2}_{ren} \le g^4 b + \frac{1}{2} g^2 C(T)$

$$< g^4 b - \frac{1}{2} g^2 C(T$$

↙ ground state energy

$$\underline{\text{s-}\lim_{\varepsilon \to 0} e^{-T(H - E(g^2))} = e^{-\exists H_\infty}}$$ ある部分列に存在する

∵ $-g^4 b \quad \frac{1}{2} g^2 C(T) \le E(g^2) - E_{ren} \le g^4 b + \frac{1}{2} g^2 C(T)$

$$H \quad E(g^2) = H \quad E_{ren} + \boxed{E_{ren} - E(g^2)}$$