

# Mathematical Aspects of Quantum Fields and Related Topics

### Akito Suzuki (Shinshu University)

### Space-time continuous limits of disvrete-time quantum walks

We derive Dirac equations from one-dimensional discrete-time quantum walks in a space-time continuous limit with mathematical rigor. This work is in collaboration with M. Maeda.

### Yosuke Kawamoto (Fukuoka Dental College)

# Stochastic analysis on infinite dimensional stochastic differential equations related to random matrices

We consider diffusions with infinitely many particles related to random point fields such as sine, Airy and Bessel random point fields. These dynamics are constructed by Dirichlet form theory, and represented as infinite dimensional stochastic differential equations. In this talk I will talk about convergence theorems of these dynamics and related topics.

### Zied Ammari (Université de Rennes 1)

### Classical electrodynamics as an effective theory of quantum electrodynamics

In this talk, I will discuss the approximation of Quantum Electrodynamics (QED) by means of particle-field classical electrodynamics in the effective regime  $\hbar \to 0$ . Several consequences related to this Bohr's correspondence principle will be presented as well as some open questions. The talk is based on an ongoing research project in collaboration with Marco Falconi and Fumio Hiroshima.

# Nobuhiro Asai (Aichi University of Education)

#### Deformed free Fock spaces and Meixner distributions.

In this talk, we shall consider the probability distribution of operators on the deformed free Fock space in a sense. This talk is based upon the joint work with H. Yoshida (Ochanomizu University).

### Hayato Saigo (Nagahama Institute of Bio-Science and Technology)

### Space-homogeneous quantum walks on Z from the viewpoint of complex analysis

The subject of this talk is quantum walks, which are expected to simulate several kinds of quantum dynamical systems. We define analyticity for quantum walks on Z. Almost all the quantum walks on Z which have been already studied are analytic. In the framework of analytic quantum walks, we can enlarge the theory of quantum walks. We obtain not only several generalizations of known results, but also new types of theorems. We will talk about some of the results including the theorem

that every analytic space-homogeneous quantum walk on Z is essentially a composite of shift operators and continuous-time analytic space-homogeneous quantum walks, and that the weak limit distribution exists for analytic space-homogeneous quantum walks on Z.

### Itaru Sasaki (Shinshu University)

# Regular Bogoliubov transformations

A Bogoliubov transformation is a basic tool to analyze quantum field Hamiltonians. It is believed that a Hamiltonian with a quadratic interaction of creation and annihilation operators can be diagonalized by a Bogoliubov transformation. In most studies, the Bogoliubov transformations are constructed through a scattering theory, and it required a regularity on coupling functions. In this talk, we provide a general method for constructing regular Bogoliubov transformations which diagonalize a general class of quadratic Hamiltonians. Our construction is algebraic and independent of spectral properties of the free Hamiltonian. This is a joint work with Y. Matsuzawa and K. Usami.

### Noriaki Teranishi (Hokkaido University)

### On the existence of time operators.

We construct a time operator of a self-adjoint operator with an infinite dimensional CCR-domain. Recently, a time operator of an unitary operator is defined. We show that there exists a time operator of any non-trivial unitary operator.

### Fumio Hiroshima (Kyushu University)

### Positivity improving and spatial decays of bound states in quantum field theory

We discuss spatial decays of bound states of Hamiltonians in quantum field theory. We show both of an upper and a lower bound of a bound state with respect to x.

# Izumi Ojima (Research Origin for Dressed Photon)

### Categorical formulation of quantum field theory

On the basis of Saigo's proposal of "category of mobility", a reformulation of quantum field theory will be attempted so as to incorporate both the on-shell and off-shell aspects required by the dressed photon physics.

### Seiichiro Kusuoka (Kyoto University)

# Invariant measure and flow associated to the $\Phi$ 4-quantum field model on the three-dimensional torus

We consider the invariant measure and flow of the  $\Phi$ 4-model on the three-dimensional torus, which

appears in the quantum field theory. By virtue of Hairer's breakthrough, such a nonlinear stochastic partial differential equation became solvable and is studied as a hot topic. In the talk, we apply the paracontrolled calculus and directly construct the global solution and the invariant measure by using the invariant measures of approximation equations and showing the tightness of associated processes. This is a joint work with Sergio Albeverio.

### Hirofumi Osada (Kyushu University)

### Diffusion in Coulomb environment and a phase transition

We consider homogenization of a particle in a Coulomb environment. We first consider a d-dimensional lattice such that each site is equally charged. Then we put a charged Brownian particle effected by the Coulomb potential with inverse temperature  $\beta$ , and take a diffusive scaling limit (homogenization). The limit dynamics becomes a Brownian motion whose constant coefficient is called effective conductivity. The effective conductivity is always positive definite in this case. We next remove a finite number of sites. We show a phase transition of the effective conductivity occurs concerning inverse temperature if dimension equals two. On the other hand, in  $d \geq 3$ , the limit is always non-degenerated. We also identify the limit dynamics for d = 2.

### Kenichi Ito (The University of Tokyo)

### Zeroth order conjugate operator in N-body Schrödinger operators

We prove Rellich's theorem and LAP for the N-body Schrödinger operator. The proofs consist of systematic use of commutators with zeroth order conjugate operator, unlike the standard first order one in the Mourre theory. This talk is based on a joint work with T. Adachi, K. Itakura and E. Skibsted.

### Hiroyuki Ochiai (Kyushu University)

### Symmetry raised from dressed photon

We summarize the symmetry on spaces related with off-shell science and dressed photons. We also add a remark on the paper by Sakuma - Ojima - Ohtsu from the invariant theory point of view.

### Yuta Aihara (Hokkaido University)

### On a General Class of Exterior Differential Operators in Boson-Fermion Fock Spaces

The notion of exterior differential operators in Boson-Fermion Fock spaces was introduced and studied by Professor Arai in the 1990s in relation to quantum field theory. In this study, integration by parts formulae for the polynomial type of functionals play an important role. For further development

of the study, we derive integration by parts formulae for a general class of functionals. Then, we apply these formulae to the definition of a class of generalized exterior differential operators in Boson-Fermion Fock spaces. And we see fundamental properties of these operators such as the de Rahm-Hodge-Kodaira decomposition.

### Asao Arai (Hokkaido University)

### Singular Bogoliubov transformations

We introduce a concept of singular Bogoliubov transformation on the abstract boson Fock space and construct a representation of canonical commutation relations (CCRs) which is inequivalent to any direct sum of the Fock representation. Sufficient conditions for the representation to be irreducible are formulated. Moreover, an example of such representations of CCRs is given.

### Keiji Nakatsugawa (Hokkaido University)

### Time Operators and Time Crystals in Ring Systems

In the standard formulation of quantum mechanics and quantum field theory, time is not an observable but merely a parameter. There are two open problems that can promote time to an observable: 1) how to define self-adjoint time operators and 2) how to obtain quantum systems called "time crystals". In this talk, we use a generalized commutation relation called the generalized weak Weyl relation to solve these problems in ring systems with macroscopic wave functions, such as charge density waves and superconductors. We surmise that time operators and time crystals are closely interrelated and also that topology change is important when time promotes from a parameter to an observable.

### Kazuya Okamura (Research Origin for Dressed Photon)

#### Towards new uncertainty relations

We show uncertainty relations for several errors and disturbances.

### Fumihiko Nakano (Gakushuin University)

### Asymptotic behavior of eigenfunctions of 1d random Schrödinger operators

We consider the 1-dimensional random Schrödinger operator with the coupling constant decaying along the system size. This model has (i) extended, (ii) critical, and (iii) localized phases, depending on the decay rate. We study the scaling limit of the measure associated to the eigenfunctions, which shows different behavior in those three cases.