Mathematical Aspects of Quantum Fields and Related Topics

July 2-4 , 2018

Research Institute for Mathematical Sciences
Kyoto University, Kyoto, Japan

Tomohiro Kanda (Kyushu University)

A model of Josephson junctions on boson systems - currents and entropy production rate -

Abstract

Non-equilibrium steady states (NESS), in the sense of D. Ruelle, of Boson systems with Bose–Einstein condensation (BEC) are investigated with the aid of the C^*-algebraic method. The model consists of a quantum particle and several bosonic reservoirs. We show that the mean entropy production rate is strictly positive, independent of phase differences provided that the temperatures or the chemical potentials of reservoirs are different. Moreover, Josephson currents occur without entropy production, if the temperatures and the chemical potentials of reservoirs are identical.

Yohei Kashima (Osaka University)

Methods of improving correction term estimates in the BCS model with imaginary magnetic field

Abstract

It has been shown that under the imaginary magnetic field the BCS model exhibits phase transitions characterized by spontaneous symmetry breaking and off-diagonal long range order even in high temperature, weak coupling regimes. The main body of the proof consists of multi-scale estimations of the correction term left after extracting the reference term from the full Grassmann integral formulation. The goal of the multi-scale estimations is to prove that the correction term is analytically controllable by keeping the coupling constant large enough to ensure the solvability of the gap equation. While the analyticity of such a term with the coupling constant can be proven by a routine, whether the allowed magnitude of the coupling constant can exceed sufficient values for the solvability of the gap equation is an essential problem. In this talk I will explain how this goal is achieved by focusing on the technical core of the multi-scale analysis on the Grassmann algebra.

Mi Ra Lee (Chungbuk National University)

One-parameter groups involving Bogoliubov and quantum Girsanov transforms

Abstract

We study a Lie group of one-parameter transformations acting on white noise functionalsassociated with the Lie algebra generated by the scalar, annihilation operator, creation operator,conservation, generalized Gross Laplacian and its adjoint operator.The one-parameter groups are involving the Bogoliubov and quantum Girsanov transforms, Fourier-Gauss and Fourier-Mehler transforms and Weyl transform.We examine the unitarity conditions for the one- parameter groups.
This talk is based on a joint work with Un Cig Ji.

Un Cig Ji (Chungbuk National University)

Anticipating stochastic integrals for Levy processes

Abstract

In this talk, based on the quantum decomposition of a Levy process, we study anticipating stochastic integrals for a Levy process. For our purpose, we first construct the admissible Levy white noise functionals and study the annihilation, creation and conservation operators as basic operators on Boson Fock space. Then we study the quantum decomposition of a Levy process and as an application, we study anticipating stochastic integrals for Levy processes.

Takahiro Hasebe (Hokkaido University)

Unimodality for classical and free Brownian motions with initial distributions

Abstract

Roughly, unimodality is the property that the density function of a probability measure attains its maximum at a unique point. We investigate how the unimodality of Brownian motion with initial distribution depends on time. In particular, we will discuss the case when the initial distribution is not unimodal. We consider a similar problem for free Brownian motion in free probability.

Serge Richard (Nagoya University)

Spectral and scattering theory for Schrödinger operators on perturbed topological crystals

Abstract

During this talk we shall present some results on the spectral and the scattering theory of Schrödinger operators acting on perturbed periodic discrete graphs. The general setting of crystal lattices will first be introduced. Then, perturbations of two types will be considered: either a multiplication operator by a short-range or a long-range function, or a short-range type modification of the measure defined on the vertices and on the edges of the graph. Mourre theory is used for describing the nature of the spectrum of the underlying operators. For short-range perturbations, existence and asymptotic completeness of local wave operators will also be proved.

Fumio Hiroshima (Kyushu University)

Integral kernel of semigroup generated by a model in QFT

Abstract

The Nelson Hamiltonian in QFT is defined by Schrödinger operator $H_p = -\Delta/2 + V$ in $L^2(R^d_x)$ linearly coupled to a scalar bose field $H_i = \phi(\varphi(\cdot - x))$ with UV cutoff function φ. Then H can be defined as a self-adjoint operator on $L^2(R^d) \otimes F$ as

$$H = H_p \otimes 1 + H_i + 1 \otimes H_f,$$

where F denotes a boson Fock space and H_f the free field operator on F defined by the second quantization of dispersion relation $\omega(k) = \sqrt{|k|^2 + \nu^2}$. Under the identification $L^2(R^d) \otimes F \cong L^2(R^d; F)$, the heat semigroup generated by H can be realized in terms of Brownian motion $(B_t)_{t \geq 0}$ on the wiener space $(\mathcal{X}, \mathcal{F}, W^x)$ as

$$(F, e^{-tH} G) = \int_{R^d} dx E^x_W \left[\left(F(B_0), J_0^* e^{-\phi_E(\int_0^t \varphi(\cdot - B_s) ds)} J_t G(B_t) \right)_F \right].$$

In the case of massive: $\nu > 0$, it can be easily seen that the kernel operator

$$J_0^* e^{-\phi_E(\int_0^t \varphi(\cdot - B_s) ds)} J_t$$

is a bounded operator on F for each $(x, w) \in R^d \times \mathcal{X}$ by the hypercontractivity of e^{-tH_f}. In the case of massless: $\nu = 0$, we can also show the boundedness by using the Baker-Campbell-Hausdorff formula. By using the boundedness of $\| J_0^* e^{-\phi_E(\int_0^t \varphi(\cdot - B_s) ds)} J_t \|$ we discuss (1) spatial decay of bound states of H, (2) super-exponential decay of the ground state of H and (3) stochastic UV renormalization of H.

Jérémy Faupin (Université de Lorraine)

Scattering theory for dissipative quantum systems

Abstract

In this talk, we will consider an abstract pseudo-hamiltonian given by a dissipative operator of the form $H = H_V - iC^*C$, where $H_V = H_0 + V$ is self-adjoint and C is a bounded operator. Such operators are frequently used to study scattering theory for dissipative quantum systems. We will recall conditions implying existence of the wave operators associated to H and H_0, and we will see that they are asymptotically complete if and only if H does not have spectral singularities on the real axis. For Schrödinger operators, the spectral singularities correspond to real resonances. This is joint work with Jürg Fröhlich.

Asao Arai (Hokkaido University)

Representation of canonical commutation relations associated with the Casimir effect

Abstract

It is shown that a very singular representation of canonical commutation relations (CCR), which is inequivalent to the Fock representation of CCR, is associated with the Casimir effect and how the Casimir force can be derived from the representation.

Itaru Sasaki (Shinshu University)

Existence of ground state of the model of a massless charged particle interacting with the quantized radiation field

Abstract

We consider the massless relativistic Pauli-Fierz model which describes the massless charged particle interacting with a quantized radiation field. The Hamiltonian of the model have the form $H = |p - eA(x)| + H_f + V(x)$ where p is the momentum of the particle, $A(x)$ is the quantized vector potential, H_f is the free energy of the photon and $V(x)$ is the external potential. We show the existence of ground state of this model under suitable condition for V.

Thomas Norman Dam (Aarthus University)

Non-renormalizability of Spin-Boson type models

Abstract

The Spin-Boson type models is a well known and often used family of models for a two level system interacting with a boson field a Boson field. Write Δ for the energy difference in the two level system.

In order to make mathematical sense of the model one introduces an ultraviolet cutoff in the interaction between the field and the two level system. As the choice of cutoff is arbitrary one would want to remove it by taking limits just as in the Nelson model. We will present a new result, which as a corollary shows, that if an asymptotic limit exists for one choice of Δ in some reasonable sense then a limit exists for all Δ and it is independent of Δ. In particular there is no physically reasonable way to renormalise the Spin-Boson model. We emphasize that our results holds for both the massless and the massive models and for every coupling strength.

This is joint work with Jacob Schach Møller.

Cid Reyes Bustos (Kyushu University)

Spectral degeneracies in the asymmetric quantum Rabi model

Abstract

The *quantum Rabi model* (QRM) is the simplest model of quantum optics used to describe the interaction of matter and light. The Hamiltonian of the *asymmetric quantum Rabi model* (AQRM) is given by

$$H^{\epsilon}_{\text{Rabi}} := a^{\dagger}a + \Delta\sigma_z + g(a + a^{\dagger})\sigma_x + \epsilon\sigma_x,$$

it is obtained by adding the symmetry-breaking term $\epsilon\sigma_x$ to the Hamiltonian of the quantum Rabi model. The QRM ($\epsilon = 0$) has a natural $\mathbb{Z}/2\mathbb{Z}$ symmetry that gives rise to degeneracies on the spectrum, such a symmetry is lost in the general AQRM case.

In this talk we give a description of the degeneracy structure of the AQRM for the general $\epsilon = \ell/2 \in \frac{1}{2}\mathbb{Z}$ case. The presence of level crossings on the spectrum of the AQRM for the parameter $\epsilon = \frac{1}{2}$ was first observed numerically by Li and Batchelor (2016). Soon after that, Wakayama (2016) formally proved the degeneracy for this case and conjectured the result for the general case. We prove the conjectures, showing the presence of degeneracies for the case $\epsilon = \ell/2 \in \frac{1}{2}\mathbb{Z}$. We complete the picture by characterizing the eigensolutions and parameters (g, Δ) for which degeneracies appear.

The G-function of the AQRM was described by Daniel Braak in 2011 in his seminal study of the integrability of the quantum Rabi model. The zeros of the G-function correspond to the so-called regular spectrum of the AQRM. We show that the remaining eigenvalues, forming the exceptional spectrum, are partially described in the poles of the G-function. This description, exhibits the G-function as the spectral determinant corresponding to the spectral zeta function of the AQRM up to a non-vanishing entire factor.

This talk is based on a joint research with Kazufumi Kimoto and Masato Wakayama.

References

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Izumi Ojima

Micro-Macro duality for inductions/ reductions

Abstract

Paradoxical appearance of negative metrics in the processes of emergences will be analyzed from the viewpoint of Morse theory, induced representations and of imprimitivity systems.

Kazuyuki Wada (National Institute of Technology, Hachinohe College)

An asymptotic velocity operator for a class of long-range type quantum walks

Abstract

We consider 1-dimensional quantum walks. One of our interest in quantum walks is to determine the probability distribution of an asymptotic velocity of a quantum walker. To do it, it is important to construct an asymptotic velocity operator. If a coin operator C is position independent, an explicit form of the asymptotic velocity operator is known [Grimmett, Janson and Scudo, 2004]. If C satisfies a "short-range condition", the asymptotic velocity operator is constructed through wave operators [Suzuki, 2016][Richard, Suzuki and Tiedra de Aldecoa, 2018]. If C satisfies a "long-range condition", it is not trivial to construct the asymptotic velocity operator since the standard wave operator does not exist in general. In this talk, we construct the asymptotic velocity operator for a class of quantum walks whose coin operator C satisfies a long-range condition.

Kazuya Okamura (Nagoya University)

A $C*$-algebraic approach to quantum measurement

Abstract

We propose a new framework for quantum measurement theory in the $C*$-algebraic setting. Different from the von Neumann algebraic setting, we have to describe processes of measurements consistent with transitions among sectors. Processes of measurements both in the Schrödinger picture and in the Heisenberg one are examined herein. We analyze completely positive instrument defined on $C*$-algebras in the Schrödinger picture and introduce the notion of measuring scheme in the Heisenberg picture.

Akito Suzuki (Shinshu University)

Supersymmetric quantum walks

Abstract

Chiral symmetric quantum walks exhibit supersymmetry. We define the Witten index for such a walk, provide an index formula, and demonstrate how to calculate the indices for several concrete models.