In this talk I will give an overview of the explicit computation of the heat kernel of the quantum Rabi model (QRM) by means of the Trotter-Kato product formula, recently obtained by the speaker and Masato Wakayama, and some of its consequences. In addition, certain aspects of the computations are reinterpreted to allow a generalization to other related models (asymmetric QRM, Dicke model, etc). For instance, the infinite series appearing in the expression of the heat kernel can be described in terms of orbits of the action of the infinite symmetric group (defined by direct limit) on certain groups determined by the models studied.