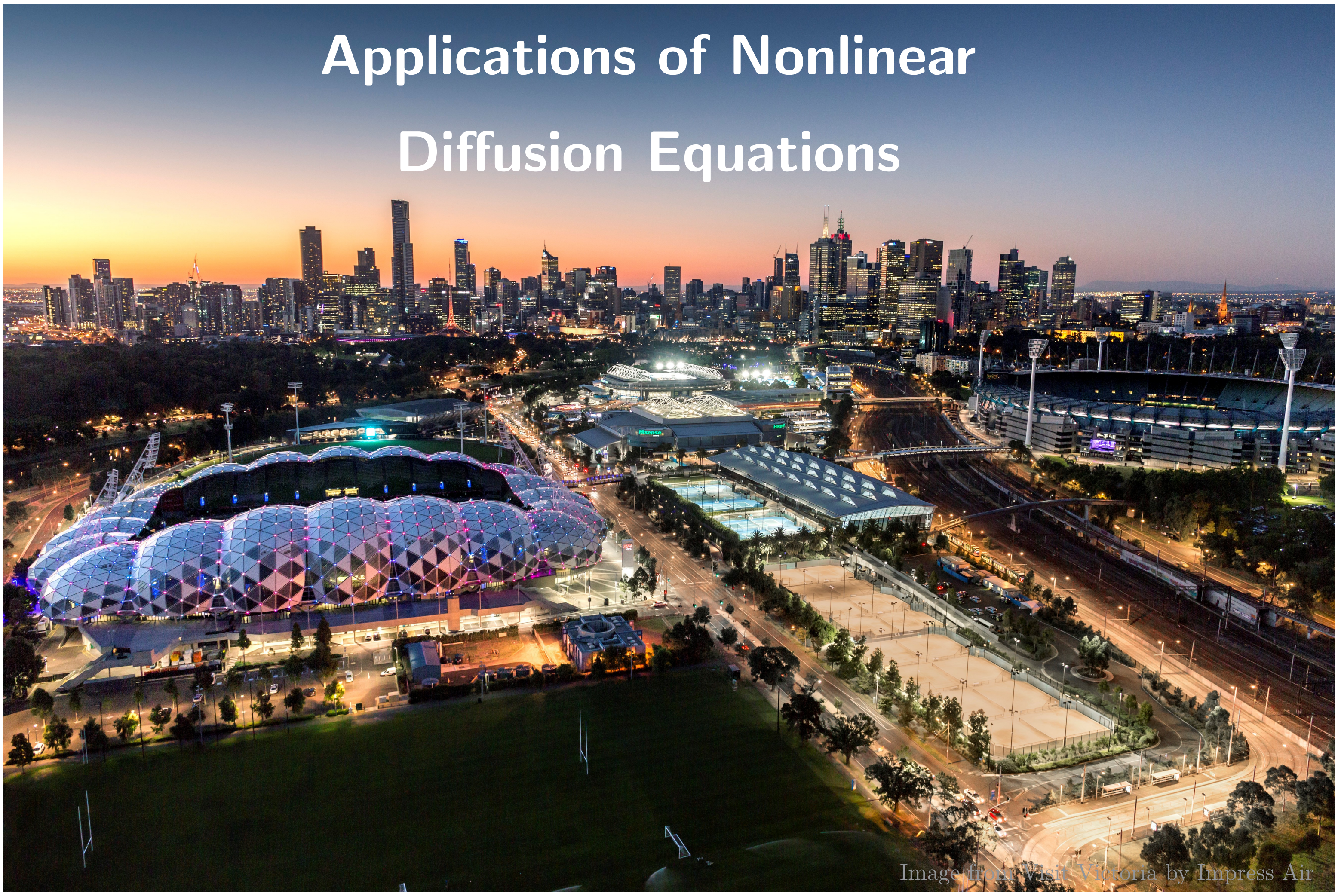
Applications of Nonlinear Diffusion Equations



Melbourne, June 19-21, 2019

Book of Abstracts

Sponsors

Information

Conference venue

La Trobe University's City Campus, Level 20, 360 Collins St, Melbourne, Victoria.

Delegates will be able to register from 8.45 – 9.15am on Wednesday 19^{th} at the workshop venue.

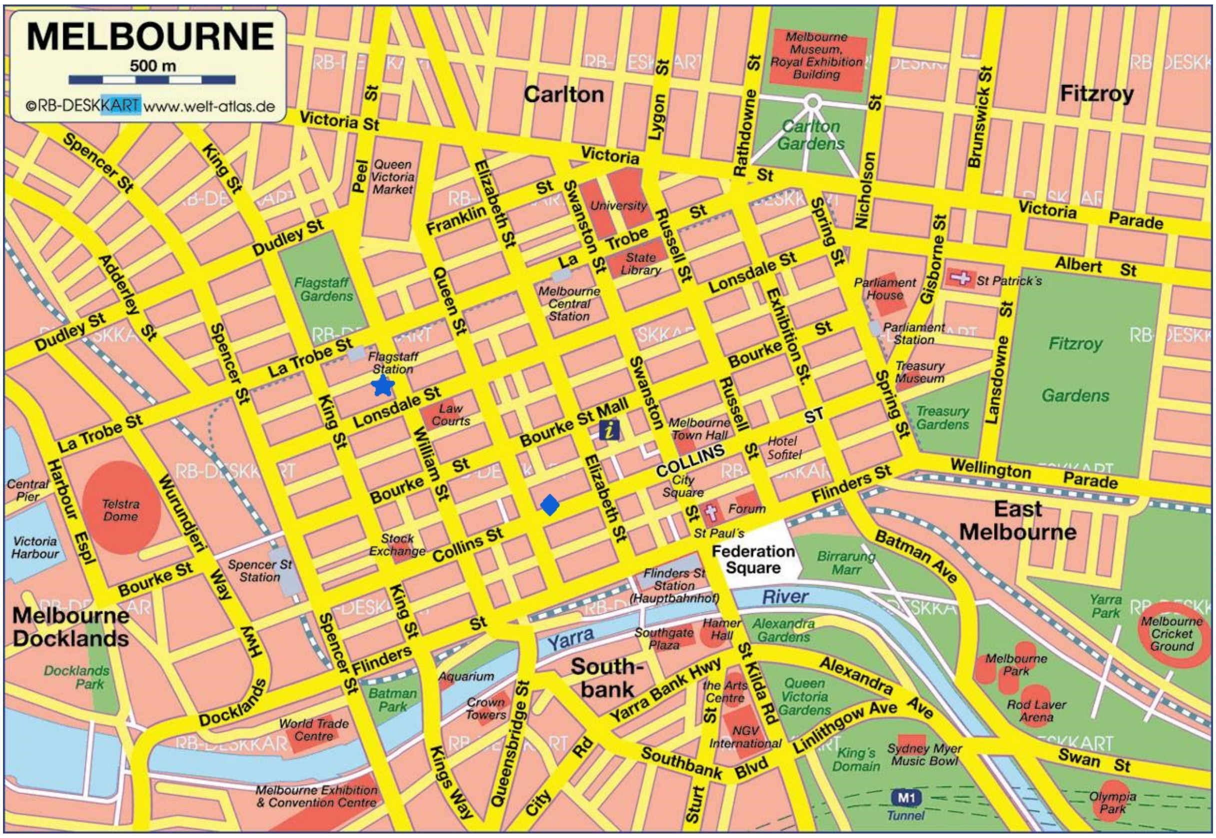
Light morning and afternoon teas will be provided on Level 20 of the Collins St campus.

Lunches will not be provided at the workshop. There are many lunch options nearby, particularly towards Elizabeth St on Little Collins St (behind the workshop venue), or alternatively towards Flinders Lane, DeGraves St, and The Block Arcade.

There is plenty of allocated **collaborative time** during the workshop. Delegates are welcome to use the spaces on Level 20.

There will be a **retirement dinner** for Professor Philip Broadbridge at 6.30pm on Thursday 20^{th} June at the Metropolitan Hotel, 263 William St, Melbourne. The Hotel is a 12 minute walk from the workshop venue.

In the map below, the workshop venue is marked with a blue diamond and the dinner venue is marked with a blue star.



Presentations

All contributed talks are of 15 minutes duration. The session chair will give you a signal when you have 5 minutes remaining (at the 10 minute mark). There will be an additional 5 minutes for questions and discussion. Please do not exceed your time.

The room will be equipped with a computer running Windows, with mouse, keyboard, USB ports, and internet connection.

We strongly encourage you to bring your talk in the form of a PDF document, preferably on a USB storage device. Please make sure that your talk is copied onto the desktop computer *well in advance* of your talk. The desktop in the talk room can display Powerpoint documents but we cannot guarantee that all fonts are available. Note that postscript files cannot be displayed. If you require access to other software packages or alternative audio-visual equipment please talk to the workshop organisers well in advance to see whether it can be arranged. If you wish to use your own laptop, please be sure to bring your own adapters if necessary.

Internet access

If you are visiting from a participating 'eduroam' institution you can connect to the eduroam wireless network using your login credentials (username and password) and security settings from your home institution.

Non-eduroam guest access to the wireless network will also be available; check at the Registration Desk for more details.

Wednesday, 19^{th} June

8:45	Registration
9:15	Welcome
9:20	Invited speaker: Maria Clara Nucci A 25-year review of the role of heir-equations
10:00	Ngamta Thamwattana Modelling dye-sensitized solar cells
10:20	Morning tea
10:50	Matthew Simpson A free boundary nonlinear diffusion model of epithelial dynamics
11:10	Scott McCue Similarity solutions of the porous medium equation and their application to cell migration assays
11:30	Graeme Wake Cell population growth models: Beating the big C
11:50	Collaborative time
12:30	Lunch
1:30	Invited speaker: Kenji Kajiwara The Burgers-type equations in the deformation theory of curves
2:10	Valentina Wheeler Mathematics of bushfires: Link to geometric analysis and curvature flows
2:30	James McCoy A rigidity theorem for ideal surfaces with flat boundary
2:50	Collaborative time
3:40	Afternoon tea
4:00	Yasuhide Fukumoto A finite difference scheme for the Richards equation under variable-flux boundary conditions
4:20	Mark McGuinness Great balls of fire
4:40	Elizabeth L. Mansfield Noether's Theorem 100 years later

Thursday, 20^{th} June

9:00	Invited speaker: Roman Cherniha A hunter-gatherer-farmer population model: Lie symmetries, exact solutions and their interpretation
9:40	R. Joel Moitsheki Applied differential equations: Symmetry solutions
10:00	Nalini Joshi Nonlinear elliptic-difference-type equations
10:20	Morning tea
10:50	Yvonne Stokes Oocyte fertilisation, chemical signalling and waves
11:10	Bronwyn Hajek Analytic solutions for calcium fertilisation waves on amphibian eggs
11:30	Mary Myerscough A structured population model for lipid accumulation in macrophages
11:50	Collaborative time
12:30	Lunch
1:30	Invited speaker: Maureen Edwards Lie symmetries and solutions of nonlinear differential equations: A talk in two parts
2:10	Timothy Marchant Dispersive shock waves governed by the Whitham equation and their stability
2:30	Tony Roberts Attraction to self-similarity in nonlinear lubrication, and in stochastic systems
2:50	Collaborative time
3:40	Afternoon tea
4:00	Invited speaker: Mary Pugh Smectic electroconvection, Poisson-Nernst-Planck equations, and oddities in time-stepping
4:40	Geoff Prince The Frobenius integrability of the KdV equation and its soliton
5:00	Peter Tritscher Series solution for a two spatial-dimension canonical geometry Stefan problem

6:30	Philip Broadbridge retirement dinner The Metropolitan Hotel, 263 William Street, Melbourne

Friday, 21^{st} June

9:00	Invited speaker: Masato Wakayama Heat kernel of the quantum Rabi model
9:40	Peter Clarkson Rational solutions of Painleve equations
10:00	Andriy Olenko Random spherical hyperbolic diffusion
10:20	Morning tea
10:50	Neville Fowkes Modification patterns in germinating barley
11:10	Laura Karantgis Modelling rainfall induced landslides with analytical and numerical methods
11:30	Antoinette Tordesillas Fusing complex networks and AI to characterise diffusion of kinematic information for early prediction of landslides
11:50	Collaborative time
12:50	Lunch
1.50	Xiaoping Lu Option pricing with transaction costs under Heston stochastic volatility
2:10	Dimetre Triadis Exact solutions for groundwater infiltration subject to surface ponding
2:30	Kerry Landman and Bob Anderssen ODE to the Broad-Bridge that is Phil
2:50	Afternoon tea
3.20	Invited speaker: Philip Broadbridge Conditionally integrable nonlinear diffusion models
4.00	Collaborative time & Workshop end

Keynote Speakers

(listed alphabetically)

Professor Philip Broadbridge

La Trobe University, University of Wollongong and IMI Kyushu University

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Conditionally integrable nonlinear diffusion models

This considers useful nonlinear PDE systems whose symmetry group contains a general solution of a linear PDE, but with a lower dimensional graph space. The systems include

(a) A class of reaction-diffusion equations in n dimensions, requiring a single relationship between nonlinear diffusivity and nonlinear reaction (with a recent application to design of fishing exclusion zones);

(b) A complex-valued Madelüng fluid picture of quantum mechanics, that reduces to the Schrödinger equation by the Hopf-Cole transformation of a Burgers fluid. This provides a dynamical model of the quantum measurement problem;

(c) 2D electron gas diffusion with reciprocal diffusivity $D = \frac{1}{u}$.

Biography: Phil Broadbridge was educated at University of Adelaide (Maths Physics) and University of Tasmania (Education). He has previously worked as a high school teacher and as a CSIRO Research Scientist. He held Level E positions for 28 years at Wollongong, Delaware, Melbourne and La Trobe. In 2008, as Director of the Australian Mathematical Sciences Institute, he received the 2008 Fast Thinking / Open Universities award for science. He has published over 130 papers in 34 different journals, on a range of topics including quantum physics, hydrology, thermodynamics, population genetics, metal surface evolution and solution methods for nonlinear partial differential equations. He has been on editorial boards of eight different journals, currently including Proceedings of the Royal Society of London Series A, Applied Mathematical Modelling, Entropy, ANZIAM Journal and International Journal of Industrial Mathematics.

Professor Roman Cherniha

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A hunter-gatherer–farmer population model: Lie symmetries, exact solutions and their interpretation

The Lie and Q-conditional symmetry classification of the known three-component reaction-diffusion system [K.Aoki et al, Theor. Popul. Biol. 50, 1-17 (1996)] modelling the spread of an initially localized population of farmers into a region occupied by hunter-gatherers is derived. The Lie and conditional symmetries obtained for reducing the system in question to systems of ODEs and constructing exact solutions are applied. Several highly nontrivial exact solutions, including traveling fronts, are found, their properties are identified and biological interpretation is discussed.

The talk is based on new unpublished results and the recent paper by R.Cherniha & V.Davydovych, Euro. Jour. Appl. Math.(2019) doi:10.1017/S0956792518000104

Biography: Roman M. Cherniha is a professor at the Institute of Mathematics, National Academy of Sciences, Kyiv, Ukraine.

Roman Cherniha graduated (with honours) in Mathematics from the Taras Shevchenko Kyiv State University (1981), defended PhD dissertation (1987) and habilitation (2003) at the Institute of Mathematics, NAS of Ukraine. In 2012, he got the academic degree Professor in Mathematics from the government of Ukraine.

During his early stage career, Roman got a substantial experience on the field of applied mathematics and physics at the Institute of Technical Heat Physics, Academy of Sciences of Ukraine. Since 1992, he has a permanent research position at the Institute of Mathematics. He spent a few years abroad working at the Universite Henri Poincare Nancy I (a temporary CNRS research position) and the University of Nottingham (Marie Curie Research Fellow).

Roman has a wide range of research interests including: Non-linear partial differential equations (especially reaction-diffusion equations): Lie and conditional symmetries, exact solutions and their properties; Development of new methods for analytical solving non-linear PDEs; Application of modern methods for analytical solving nonlinear boundary-value problems, arising in real world application; Analytical and numerical solving boundary-value problems with free boundaries; Development of mathematical models describing the specific processes arising in physics, biology and medicine.

Professor Cherniha has also a substantial pedagogical experience obtained, in particular, at the National University ‘Kyiv Mohyla Academy, where he was professor in Mathematics for five years. During the last decade, he was the supervisor for five PhD students. He is the author of over 100 scientific papers (66 of them are indexed by the Scopus database) and two monographs published in prestigious publishing houses Springer and CRC Press. Professor Cherniha has also acted as a referee for several international scientific journals, and is an editorial board member of the journal Symmetry.

Dr Maureen Edwards

University of Wollongong

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Lie symmetries and solutions of nonlinear differential equations: a talk in two parts

The Lie symmetry properties of two biologically motivated problems are considered.

Autonomous ODE models for microbial growth in a closed environment give solutions that can only grow or decay monotonically or asymptote, never capturing the mortality phase in a typical microbial growth curve. The non-autonomous von Bertalanffy equation is proposed as a model of the interaction of the current size of a population with the environment in which it is living. The relationship between the introduced non-autonomous terms is explored through Lie symmetry analysis.

Biological invasion, such as the spread of a virus on a plant leaf, has a well defined progressing compactly supported spatial structure. The compactly supported Pattle solution for nonlinear diffusion equations can be explained using Lie symmetry analysis. The behaviour of other choices of nonlinear diffusion and the addition of reaction terms which retain a compactly supported structure is considered.

This is joint work with with Bob Anderssen and Bronwyn Hajek.

Biography: Maureen Edwards completed her BMath(Honours) at the University of Wollongong, followed by her PhD in Applied Mathematics under the supervision of Phil Broadbridge. She was appointed to an academic position at the University of Wollongong in 1994 while completing her PhD. Her research interests include mathematical models motivated by biological invasion and population dynamics, and the application of Lie symmetry techniques to classes of nonlinear differential equations. Maureen is currently Deputy Head of the School of Mathematics and Applied Statistics.

Professor Kenji Kajiwara

Kyushu University

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The Burgers-type equations in the deformation theory of curves

It is well-known that the plane/space curves in the Euclidean geometry admit deformations governed by certain integrable systems. The famous model is the so-called binormal deformation of the space curves described by the nonlinear Schrdinger equation, which serves as the physical model of the vortex filament. Also, the modified KdV equation describes the isoperimetric deformation of the plane curves. The families of curves obtained as the rigid motions described by the traveling wave solutions of those equations form important classes, namely, the Kirchoff rod and the Euler's Elastica, respectively. Considering the Klein geometry gives rise to various integrable deformations according to the group actions. In particular, in the context of the similarity geometry, the integrable deformation of the plane curves is described by the Burgers equation, and its rigid motion includes an interesting class of curves, called the log-aesthetic curves (LAC). LAC is proposed in the area of industrial design as a family of curves that the car designers regard as "aesthetic." Based on this observation, we construct an integrable discretization of LAC which gives a fast and high-quality implementation of LAC. We also formulate a space curve extension of LAC which is characterized as the rigid motion of the integrable deformation of space curves under the similarity geometry which is governed by the coupled system of the third-order Burgers equation and the modified KdV equation. We also present the explicit solutions and the variational formulation of those curves.

Biography: Kenji Kajiwara has been a Professor of the Institute of Mathematics for Industry (IMI), Kyushu University since its foundation in 2011. He has been Deputy Director of IMI since 2018, and is mainly in charge of MEXT Joint Usage/Research Center activities and international activities such as the Asia Pacific Consortium of Mathematics for Industry. In particular, he has managed the Australia Branch of IMI at La Trobe University since 2015 together with Professor Philip Broadbridge.

He completed an undergraduate course at the Department of Mathematical Engineering of the University of Tokyo in 1989, then Masters and Ph.D. courses at the Department of Applied Physics of the University of Tokyo in 1991 and 1994, respectively. He started his academic career at the Department of Electrical Engineering of Doshisha University in Kyoto in 1994, then moved to the Faculty of Mathematics of Kyushu University in 2001 as Associate Professor, before being promoted to Professor in 2009.

His research interests are integrable systems, integrable (discrete) differential geometry and related areas. In particular, he has contributed to the theory of (discrete) Painlev equations. Recently he is developing the theory of "aesthetic geometric shapes."

Associate Professor Maria Clara Nucci

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A 25-year review of the role of heir-equations

Heir-equations were found by iterating the nonclassical symmetry method. Apart from inheriting the same Lie symmetry algebra of the original partial differential equation, and thus yielding more (and different) symmetry solutions than expected, the heir equations are connected to conditional Lie-Bäcklund, and generalized conditional symmetries; moreover they solve the inverse problem, namely a special solution corresponds to the nonclassical symmetries. A review of 25-year work is presented, and open problems are brought forward.

Biography: Maria Clara Nucci is Associate Professor of Mathematical Physics at University of Perugia, where she graduated in Mathematics summa cum laude. From 1979 to 1981 she was the recipient of a post-graduate fellowship from the National Research Council of Italy, soon after she was confirmed as Tenured Researcher in Mathematical Physics, and has been in her current position since 2005. In 1985 she was awarded a N.A.T.O. Fellowship in Mathematics. Between 1986 and 1991 she was Visiting Assistant Professor at Georgia Institute of Technology, Atlanta (U.S.A.). She has also been invited at universities in U.K., Canada, France, Germany, Greece, and Sweden. She has presented her research at many International Congresses and Workshops. From 2000 to 2001 she was Member of the Mathematics and Computer Science Panel of Ireland. From 1995 to 2009 she was Associate Editor of Journal of Mathematical Analysis and Applications, and since 2003 has been a Member of the Editorial Board of Journal of Nonlinear Mathematical Physics.

She is author or co-author of more than 100 publications (source Scopus: documents 95, h-index 23). She has supervised students in Mathematics and Physics, and ten are among her co-authors. Through the years her research has been partially funded by Italian national grants. She has wide ranging research interests, from fluid to rigid body mechanics, from epidemiology to astrophysics, from history of mathematics to quantum mechanics. In particular, she has turned a butterfly into a tornado, connected non-local with local symmetries, unified non-classical and classical symmetries of evolution equations, quantized classical mechanics by using Noether symmetries, determined the linear equations underpinning maximally superintegrable systems. Her aspiration is to find hidden symmetries in everything.

Maria Clara Nucci's website is at:

https://protect-au.mimecast.com/s/9P96Ck81ZOtkQjq0i2JKBw?domain=dmi.unipg.it

Professor Mary Pugh

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Smectic Electroconvection, Poisson-Nernst-Planck Equations, and oddities in time-stepping

In the first part of the talk, I'll discuss electroconvection in a free submicron-thick liquid crystal film in an annular geometry. The film is flat in the x-y plane; seen from above it looks like a DVD. (Seen from above, it has two boundaries: concentric circles.) A voltage is applied across the film, from the inner boundary to the outer boundary; this voltage provides a convective forcing. The liquid crystal is in smectic-A phase, forming a nearly-perfect two-dimensional fluid because the film does not change thickness, even while flowing. Also, the inner electrode can be rotated and so the experiment can be used to study the interplay between a stabilizing force applied via the boundary (Couette shear) and convection. We present numerical simulations of special solutions such as convection cells, oscillatory convection cells, undulating convection cells, and localized vortex solutions.

This is joint work with Stephen Morris (Physics, University of Toronto).

In the second part of the talk, I'll discuss the Poisson-Nernst-Planck equations with generalized Frumkin-Butler-Volmer boundary conditions (PNP-gFBV). These equations describe ion transport with Faradaic reactions and have applications in a wide variety of fields. I'll discuss s variable-step-size semi-implicit time-stepping scheme we developed for the PNP-gFBV equations. With the method we develop, we are able to run simulations with a large range of parameters, including any value of the length scale parameter epsilon. However, as the system equilibrates, the adaptive time-stepper does not coarsen its time steps to be arbitrarily large; I'll discuss the cause of this behaviour and what it means for the numerical stability domain of the underlying solution.

This is joint work with Dave Yan and Francis Dawson (both from Electrical and Computer Engineering, University of Toronto).

Biography: Mary Pugh is an applied mathematician using modelling, nonlinear PDE, and scientific computing in fluid dynamics and conservation laws. She earned her math degrees at UC Berkeley (BSc) and the University of Chicago (MSc and PhD). She's currently a professor at the University of Toronto.

Professor Masato Wakayama

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Heat kernel of the quantum Rabi model

The quantum Rabi model describes the simplest but most significant strongly coupled quantum systems, i.e. a single bosonic mode interacts with a two-level system. It forms the basic building block of theoretical approaches to the interaction of light with matter or to electron-phonon interaction. Therefore, it is highly expected to applications to the quantum information technology. I would like to make a brief introduction to the recent theoretical and experimental progress relevant to the model, and to our study on the heat kernel and spectrum using an idea from symmetry.

Biography: Masato Wakayama completed his BS at Tokyo University of Science (1978) and got his MS and Dr. Sci. at Hiroshima University (1985). He became Professor of Mathematics, Kyushu University in 1997, held positions such as Dean of Faculty of Mathematics and Graduate School of Mathematics (2006-2010), Director of Mathematical Research Center for Industrial Technology (2010-2011), Director of Center for the Promotion of Excellence in Higher Education (2010-2011), and was inaugurated as Senior Vice President for Education of Kyushu University in 2010 (-2014). He became Founding Director of the Institute of Mathematics for Industry (IMI), Kyushu University, at its establishment in 2011 (-2014), and at the same time became Executive Vice Dean of Faculty of Arts and Science, Kyushu University. He is the Chair of the Commission of Mathematics Innovation in MEXT, Japan, from 2011, Editor-in-Chief of International Journal of Mathematics for Industry, 2019- (Formerly, Pacific Journal of Mathematics for Industry, 2009-).

His main research area is Representation Theory and Number Theory. He has more than 100 journal-publication counts, including in Combinatorics and Mathematical Physics. He was a Visiting Fellow at Princeton University in 1995-1996, Visiting Professor at the University of Bologna (1999, 2000, 2006) and Korea Institute for Advanced Study (2001), and Distinguished Lecturer at Indiana University (2013). He served as Distinguished Professor in 2009 and is the Executive Vice President and Board Member of Kyushu University from 2014.

Masato Wakayama's website can be found at:

http://imi.kyushu-u.ac.jp/ wakayama/eprofile.html

Abstracts of contributed talks

Rational solutions of Painleve equations

Peter Clarkson
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In this I shall discuss rational solutions of Painleve equations. The six Painleve equations were derived by Painleve and his colleagues in the late 19th and early 20th centuries in a classification of second order equations whose solutions have no critical points. Painleve equations frequently arise as symmetry reductions of partial differential equations, so exact solutions of Painleve equations give rise to exact solutions of the partial differential equations. Although the general solutions of the six Painleve equations are transcendental, five of the Painleve equations possess rational solutions. These solutions are usually expressed in terms of logarithmic special polynomials that are Wronskians, often of classical orthogonal polynomials such as Hermite polynomials and Laguerre polynomials. It in known that the roots of these special polynomials are symmetric in the complex plane.

In particular, I shall discuss some recent work on rational solutions of the fourth Painleve hierarchy (with Gomez-Ullate, Grandati and Milson, arXiv:1811.09274).

Modification Patterns in Germinating Barley

Neville Fowkes
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During germination enzymes are produced that dramatically modify of the structure of the endosperm and convert starch to sucrose which is used as a food source for the growing shoot.

Experimentalists have observed two completely different modification patterns; a controversial issue in the literature. Based on an enzyme reaction, strongly non-linear diffusion model of the process we show that one of the observed patterns represents a natural propagation mode that acts as an attractor for the system. The speed of approach to this mode is strongly effected by initial conditions, a consequence of the enzymic nature of the reaction and the dramatic change of diffusivity brought about by the reaction.

The work arose out of an attempt to understand problems associated with the brewing of beer that had their origin in the malting process.

This is joint work with Ricky O'Brien.

A finite difference scheme for the Richards equation under variable-flux boundary conditions

Yasuhide Fukumoto
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The Richards equation is a degenerate nonlinear partial differential equation that models a flow through saturated/unsaturated porous media. Its numerical methods have been developed in many fields. Implicit schemes based on a backward Euler format are widely used in calculating it. However, it is difficult to obtain stability with a numerical scheme because of the strong nonlinearity and degeneracy. We establish a linearized semi-implicit finite difference scheme that is faster than backward Euler implicit schemes. We analyze the stability of this scheme by adding a small perturbation to the coefficient function of the Richards equation. It is found that there is a linear relationship between the discretization error in the L^∞-norm and the perturbation strength.
This is a collaboration with Liu Fengnan (Dalian University of Technology, China) and Xiaopeng Zhao (Jiangnan University, China).

Analytic solutions for calcium fertilisation waves on amphibian eggs

Bronwyn Hajek
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When an amphibian egg is fertilised, a wave of calcium ions travels around the surface of the egg to help prevent the entry of multiple sperm. This process can be described with a nonlinear reaction-diffusion equation with a cubic reaction term. Here, we use a particular nonclassical symmetry that gives rise to a transformation that will linearise and separate (in time and space) the reaction-diffusion equation so that analytic solutions may be constructed. We present the first analytic solutions to this 30 year old problem, demonstrating various experimentally observed phenomena, including waves and spirals.

This is joint work with Phil Broadbridge.

Nonlinear elliptic-difference-type equations

Nalini Joshi
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There is an integrable elliptic difference equation discovered by Sakai (2001), which has attracted a great deal of attention in recent times. Since its discovery, a longstanding question has been whether it is unique as the only integrable elliptic-difference-type equation. I will report on results obtained with Atkinson, Howes, Nakazono and describe two new examples of elliptic-difference equations investigated in our recent papers. One of these is new.

Modelling rainfall induced landslides with analytical and numerical methods

Laura Karantgis
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Landslide events have a devastating impact on communities and industries. Modelling these complex systems is valuable for predictive and preventative measures to reduce the impact of these events. Landslides are often caused by heavy rainfall so soil water content is important to consider when considering slope stability as it alters the soil strength.
We have constructed analytic series solutions for the phreatic free surface problem of two dimensional steady downslope saturated-unsaturated flow, with water exiting at a seepage face. This model will be used to predict the water table and flow of water through soil for varying parameters such as slope angle, length, rainfall rate, and soil type. We also investigate modelling these groundwater flow problems with a numerical method, Smoothed Particle Hydrodynamics. We obtain results for one dimensional and two dimensional infiltration problems and use analytical solutions to validate our numerical results.
We then use Smoothed Particle Hydrodynamics to model the slope failure. The results for the water table will be used to define a saturated region that considers pore water pressure and a change of the soil strength parameters of cohesion and friction angle. The results of this investigation will be compared with experimental data to validate results.

ODE to the Broad-Bridge that is Phil

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A short tribute to our Phil and a charming walk down memory lane.

Option pricing with transaction costs under Heston stochastic volatility

Xiaoping Lu
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In this work, we propose a pricing model for pricing options with transaction costs under Heston-type stochastic volatility. The resulting pricing partial differential equation (PDE) is a non-linear convection-diffusion-reaction equation with mixed derivative terms. The non-linear PDE is solved numerically by the explicit Euler method. Numerical experiments are presented to illustrate the effect of the transaction costs on option prices.

Noether's Theorem 100 years later

Elizabeth L Mansfield
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Emmy Noether's famous paper on conservation laws appeared in 1918, and there has been a celebration of the centenary of her work in many places. In this talk, I will discuss computational aspects of these theorems. I will first show how a moving frame gives insight into the solution structure of the Euler-Lagrange equations, and then indicate how her two theorems have been extended to some numerical versions of the variational problems.

Dispersive shock waves governed by the Whitham equation and their stability

Timothy Marchant
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Dispersive shock waves (DSWs), also termed undular bores in fluid mechanics, governed by the nonlocal Whitham equation are studied in order to investigate short wavelength effects that lead to peaked and cusped waves within the DSW. This is done by combining the weak nonlinearity of the Korteweg-de Vries equation with full linear dispersion relations. The dispersion relations considered are those for surface gravity waves, the intermediate long wave equation and a model dispersion relation introduced by Whitham to investigate the 120^o peaked Stokes wave of highest amplitude. A dispersive shock fitting method is used to find the leading (solitary wave) and trailing (linear wave) edges of the DSW. This method is found to produce results in excellent agreement with numerical solutions up until the lead solitary wave of the DSW reaches its highest amplitude. Numerical solutions show that the DSWs for the water wave and Whitham peaking kernels become modulationally unstable and evolve into multi-phase wavetrains after a critical amplitude which is just below the DSW of maximum amplitude.

A rigidity theorem for ideal surfaces with flat boundary

James McCoy
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We are interested in surfaces with boundary satisfying a sixth order nonlinear elliptic partial differential equation associated with extremisers of the L^2-norm of the gradient of the mean curvature. We show that such surfaces satisfying so-called 'flat boundary conditions' and small L^2-norm of the second fundamental form are necessarily planar.

This is joint work with Glen Wheeler.

Similarity solutions of the porous medium equation and their application to cell migration assays

Scott McCue
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The porous medium equation (PME) is a second-order nonlinear degenerate diffusion equation that has many applications in the physical sciences, but is also of some interest in mathematical biology. In the context of modelling cell migration assays, the PME is attractive because it allows for sharp-fronted solutions with compact support, whereas the linear diffusion equation does not. It is well known that the PME has similarity solutions of the first kind which conserve mass and approach a delta function in the limit $t \to 0^+$. These act as asymptotic solutions to the relevant Cauchy problem in the large-time limit. In contrast, hole-closing problems for the PME, for which the initial conditions are identically zero within a compact domain, are characterised by similarity solutions of the second kind, as conservation of mass no longer applies. Here the similarity solutions are relevant in the limit that the hole closes, $t \to t_c^-$. We shall summarise these issues and discuss how the PME and related reaction diffusion equations can be used to model cell migration experiments such as scratch assays, barrier assays and other wound healing assays.

This is joint work with Wang Jin and Matthew Simpson

Great Balls of Fire

Mark McGuinness
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A Surstseyan eruption is a particular kind of volcanic eruption which involves the bulk interaction of water and hot magma. Surtsey Island was born off the coast of Iceland during such an eruption process in the 1940s. I will talk about mathematical modelling of the flashing of water to steam inside a hot erupted lava ball called a Surtseyan bomb. Our overall motivation is to understand what determines whether such a bomb will fragment or just quietly fizzle out...

Partial differential equations model transient changes in temperature and pressure in Surtseyan ejecta. I describe a model that allows for the fact that pressure and temperature are coupled, and that the process is not adiabatic. A systematic reduction of the resulting coupled nonlinear partial differential equations that arise from mass, momentum and energy conservation is described.

We are able to asymptotically decouple pressure from the temperature equation. The steady-state solution of the resulting nonlinear pressure diffusion equation provides a single parametric condition for rupture of ejecta. We find that provided the permeability of the magma ball is relatively large, steam escapes rapidly enough to relieve the high pressure developed at the flashing front, so that rupture does not occur. This rupture criterion is consistent with existing field estimates of the permeability of intact Surtseyan bombs, fizzlers that have survived.

Applied Differential Equations: Symmetry Solutions

R. Joel Moitsheki
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In this talk we demonstrate the construction of group invariant (exact) solutions for equations arising in solute transport theory, heart transfer in extended surfaces and population dynamics. A comment will be made on approximate analytical solutions of models describing heat transfer in extended surfaces.

A structured population model for lipid accumulation in macrophages

Mary Myerscough
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Macrophage foam cells are typically seen in atherosclerotic plaques. These cells have accumulated so much internalised lipid that they take on a foamy appearance under the microscope. Macrophages acquire lipid, both from the cholesterol on modfied low density lipoprotein (LDL) particles (bad cholesterol) but also from ingesting other macrophages which have become apoptotic via a process of controlled cell death. We present an advective PDE model for the populations of macrophages and apoptotic cells, structured by their internalised lipid content, find steady state solutions analytically and use this model to explore the factors that contribute to plaque progression.

This is joint work with Hugh Ford and Helen Byrne.

Random spherical hyperbolic diffusion

Andriy Olenko
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We start by giving a motivation for this research and justifying the considered stochastic diffusion models for cosmic microwave background radiation studies. Then we present the exact solution in terms of a series expansion to a hyperbolic diffusion equation on the unit sphere. The Cauchy problem with random initial conditions is studied. All assumptions are stated in terms of the angular power spectrum of the initial conditions. An approximation to the solution is given and analysed by finitely truncating the series expansion. The upper bounds for the convergence rates of the approximation errors are obtained. Smoothness properties of the solution and its approximation will be discussed. We will demonstrate that the sample Holder continuity of these spherical fields is related to the decay of the angular power spectrum. Numerical studies of approximations to the solution and applications to cosmic microwave background data will be presented to illustrate the theoretical results.

The talk is based on the paper: P.Broadbridge, A.Kolesnik, N.Leonenko, A.Olenko. (2019) Random spherical hyperbolic diffusion, submitted. This research was supported under the Australian Research Council's Discovery Project DP160101366.

The Frobenius integrability of the KdV equation and its soliton

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The travelling wave solution of the Kortweg-de Vries equation can be viewed as the projection to the configuration space of the intersection of the level sets of three functions on a jet bundle. This construction is a special case of a general exterior calculus approach to travelling wave solutions which uncovers hidden structure in particular cases. It also identifies the pitfalls in the conventional approach which arbitrarily assigns values to "constants" during the integration process.

This is joint work with Naghmana Tehseen.

Attraction to self-similarity in nonlinear lubrication, and in stochastic systems

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Similarity solutions play an important role in many fields of science. Often, outstanding unresolved questions are whether a similarity solution is dynamically attractive? and if it is, to what particular solution does the system evolve? I answer these questions by recasting the nonlinear lubrication in a form to which centre manifold theory may apply. In application to a class of nonlinear fluid lubrication problems the theory constructs a similarity solution, confirms its emergence, and determines the correct solution for any compact initial condition. Analogously stochastic self-similarity in a simple model of turbulent mixing shows how anomalous fluctuations may arise in eddy diffusivities.

A free boundary nonlinear diffusion model of epithelial dynamics

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In this work we analyse a one-dimensional, cell-based mechanical model of an epithelial sheet. In the model, cells interact with their nearest neighbouring cells and move deterministically. Cells also proliferate stochastically, with the rate of proliferation specified as a function of the cell length. This mechanical model of cell dynamics gives rise to a free boundary problem. We construct a corresponding continuum-limit description where the variables in the continuum limit description are expanded in powers of the small parameter $1/N$, where N is the number of cells in the population. By carefully constructing the continuum limit description we obtain a free boundary nonlinear diffusion partial differential equation description governing the density of the cells within the evolving domain, as well as a free boundary condition that governs the evolution of the domain. We show that care must be taken to arrive at a free boundary condition that conserves mass. By comparing averaged realisations of the cell-based model with the numerical solution of the free boundary partial differential equation, we show that the new mass-conserving boundary condition enables the coarse-grained partial differential equation model to provide very accurate predictions of the behaviour of the cell-based model, including both evolution of the cell density, and the position of the free boundary, across a range of interaction potentials and proliferation functions in the cell based model.

This is joint work with Dr Andrew Parker and Professor Ruth Baker (University of Oxford).

Oocyte fertilisation, chemical signalling and waves

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It has been known for some decades that fertilisation of some amphibian and fish eggs is followed by a wave of calcium ions over the surface of the egg, which is associated with a physical change to the surface. Similar waves are seen at other stages of embryonic development. An unfertilized mammalian egg is surrounded by cumulus cells to form a cumulus-oocyte complex (COC). Just a few years ago, medical researchers at the University of Adelaide identified, for the first time, a wave-like behaviour of the cumulus cells in COCs after sperm had been added to the culture medium, believed to be a response to fertilisation. From the speed of the wave it was inferred that the cells were responding to one or more chemical signals from locations on the surface of the egg and that calcium ion concentration was the likely signal.

I will describe some ongoing modelling and experimental work being undertaken to explain the behaviour of the cumulus cells, assuming the wave is initiated at the fertilisation site. We will examine whether calcium release by either or both of the oocyte and cumulus cells is qualitatively consistent with experimental observations.

Modelling dye-sensitized solar cells

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Dye-sensitized solar cells (DSSCs) are a novel approach to the renewable energy problem, allowing sunlight conversion while reducing production costs. Electricity generation is achieved through a series of chemical reactions designed to transport electrons as a means of creating a circuit. Current modelling approach is based on the diffusion of the density of electrons in the conduction band of a DSSC's nano-porous semiconductor. In this talk, we consider non-linear diffusion equation combining the generation and the loss of the electron density as a result of dye excitation due to sunlight and electron recombination, respectively. Applying Lie symmetry analysis, we obtain new analytical solutions, which are shown to be in good agreement with full numerical solutions.

This is joint work with Ben Maldon and Maureen Edwards.

Fusing complex networks and AI to characterise diffusion of kinematic information for early prediction of landslides

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Land, rubbish and mud slides, avalanches, rockfalls, as well as slips in mines are unfortunately common place. The impact is often devastating and costly. Much earlier warning of events could allow safer practices, including evacuation or remedial actions. Geomaterials (i.e., rock, soil and snow) are at the heart of these failure events. These granular masses do not fail spontaneously. Prior to failure, they manifest measurable movements that form complex patterns. Here we combine the techniques of complex networks and AI along with fundamental knowledge of precursory dynamics of granular failure to develop a method for early prediction of field scale failure from kinematical (surface movement) data. We demonstrate this capability by identifying the location and time of failure in two large natural slopes.

Exact solutions for groundwater infiltration subject to surface ponding

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The most general integrable model for unsaturated one-dimensional groundwater infiltration governed by the Richards equation has been known since Broadbridge and White and others presented solutions in the 1980s. However, exact solutions for many simple, physically relevant boundary conditions have yet to be derived. An exact series solution for infiltration subject to ponded water at the soil surface is presented, by decomposing the soil body into saturated and unsaturated regions separated by a moving interface. The use of efficient iterative, symbolic-computation algorithms removes restrictions on the number of terms that can be obtained in the final infiltration series. We are also able to consider behaviour in the popular but subtle nonlinear delta-function diffusivity limit within a wider class of exact solutions.

Series solution for a two spatial-dimension canonical geometry Stefan problem

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Consider a large 'V' groove gouged in a larger block of material. Then this 'V' is filled with liquid of the same material, the liquid being at precisely its freezing temperature. The base material is initially at a uniform lower temperature. We wish to determine the temporal location of the freezing front and the temperature distribution in the solidified and base material. We consider early times so that the depth of the 'V' and the base thickness and width may be assumed to be of infinite extent.

This geometry allows time to be scaled into length analogous to the one spatial-dimension Stefan problem. We present some solutions derived using classic separation-of-variables. We discuss difficulties and disappointments in the implementation of these series solutions.

Cell population growth models: Beating the big C

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A model for living cell populations, which are structured by size and are undergoing growth and division simultaneously, leads to an initial boundary value problem that involves a first-order linear partial differential equation with a functional term. This is akin to the famous pantograph equation, with advanced multiplicative functionality but here in size. Here, size can be interpreted as DNA content or mass. It has been observed experimentally and shown analytically that solutions for arbitrary initial cell distributions are asymptotic, as time goes to infinity, to an attracting certain solution called the steady size distribution. The full solution to the problem for arbitrary initial distributions, however, is elusive owing to the presence of the functional term and the paucity of solution techniques for such problems. In this presentation, we derive a novel solution to the problem for arbitrary initial cell distributions. The method employed exploits the hyperbolic character of the underlying differential operator, and the advanced nature of the functional argument to reduce the problem to a sequence of simple Cauchy problems. The existence of solutions for arbitrary initial distributions is established along with uniqueness. The asymptotic relationship with the steady size distribution is established, and because the solutions is known explicitly, higher-order terms in the asymptotic form can be readily obtained. These low parameter solutions are used to underpin experimental work in developing new anti-cancer drugs.

Mathematics of Bushfires: link to geometric analysis and curvature flows

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In this talk we give a short overview of current research in mathematical modelling of bushfires with links to developments in geometric analysis. We are particularly interested in new boundary value problems in curvature flows that could answer questions related to merging fire fronts or convalescence of spot regions. The talk is design to explain the link between the two areas: bush fire modelling and curvature flows.
