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An application of a discrete fixed point theorem to the Cournot model

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An application of a discrete fixed point theorem to the Cournot model *

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Abstract

In this paper, we apply a discrete fixed point theorem of [7] to the Cournot model [1]. Then we can deal with the Cournot model where the production of the enterprises is discrete. To handle it, we define a discrete Cournot-Nash equilibrium, and prove its existence.

Keywords: Game theory, Economics, Discrete fixed point theorem, Cournot model

1 Introduction

As is well known in the game theory, Kakutani's fixed point theorem ensures the existence of a Nash equilibrium of mixed strategies. This shows that if we are concerned with a Nash equilibrium of pure strategies, it suffices to present a discrete fixed point theorem. Indeed, Sato-Kawasaki [7] provided discrete fixed point theorems, and gave a class of non-cooperative n-person games that certainly have a Nash equilibrium of pure strategies.

On the other hand, in economics, the Cournot model [1] is a well-known mar-

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ket competition model. This is a duopoly market model that each enterprise decides production and the market decides the price according to it.

In the Cournot model, if the production scale is small, the best production should be integer. For example, there are two airlines connecting city A and city B, and each airline plans how many flights run a week. In this example, the production are the number of flights run a week. To handle this situation, discrete fixed point theorems are useful tools. Thus, in this paper, we apply a discrete fixed point theorem of [7] to the Cournot model, and consider the situation where production is discrete.

Throughout this paper, i belongs to $\{1,2\}$, $\{-i\}$ means $\{1,2\} \setminus \{i\}$, C_i $(i = 1,2)$ are enterprises engaged on the market, and $q_i \in Q_i$ is production of C_i, where $Q_i := \{0,1,\ldots,m_i\}$ $(m_i \in \mathbb{N})$. The continuous extension of Q_i is designated by $\overline{Q}_i$, that is, $\overline{Q}_i = [0, m_i]$. Also $p(q)$ denotes the price of production one lot, where $q := q_1 + q_2$, and $c_i \in \mathbb{N}$ is the marginal cost, that is, enterprise C_i costs c_i for production one lot. Then C_i's profit function is

$$\pi_i(q_i, q_{-i}) = \max\{(p(q) - c_i)q_i, 0\}.$$

Each enterprise maximizes its profit.

Definition 1.1 (Cournot-Nash equilibrium) We call a pair $(q_1^*, q_2^*) \in \overline{Q}_1 \times \overline{Q}_2$ a Cournot-Nash equilibrium, if for each $i \in \{1,2\}$

$$\pi_i(q_i^*, q_{-i}^*) \geq \pi_i(q_i, q_{-i}^*), \quad \forall q_i \in \overline{Q}_i.$$

Here we denote by $\varphi(q_{-i})$ the set of best responses of enterprise C_i to q_{-i}, that is,

$$\varphi_i(q_{-i}) = \left\{ q_i \in \overline{Q}_i \,;\, \pi_i(q_i, q_{-i}) = \max_{q_i \in \overline{Q}_i} \pi_i(q_i, q_{-i}) \right\}.$$

Future, we put $\varphi(q_1, q_2) := \varphi_1(q_2) \times \varphi_2(q_1)$. Then $q^* := (q_1^*, q_2^*)$ is a Cournot-Nash equilibrium if and only if $q^* \in \varphi(q^*)$.

Definition 1.2 (Discrete Cournot-Nash equilibrium) We call a pair $(q_1^*, q_2^*) \in Q_1 \times Q_2$ a discrete Cournot-Nash equilibrium, if for each $i \in \{1,2\}$

$$\pi_i(q_i^*, q_{-i}^*) \geq \pi_i(q_i, q_{-i}^*), \quad \forall q_i \in Q_i.$$

Here we denote by $\phi(q_{-i})$ the set of best responses of enterprise C_i to q_{-i}, which is restricted to integer, that is,

$$\phi_i(q_{-i}) = \left\{ q_i \in Q_i \,;\, \pi_i(q_i, q_{-i}) = \max_{q_i \in Q_i} \pi_i(q_i, q_{-i}) \right\}.$$

Future, we put $\phi(q_1, q_2) := \phi_1(q_2) \times \phi_2(q_1)$. Then $q^* = (q_1^*, q_2^*)$ is a discrete Cournot-Nash equilibrium if and only if $q^* \in \phi(q^*)$.

In this paper, we assume the following (H1)–(H5):

(H1) $p(\cdot)$ is piecewise continuous on some open interval which contains $[0, m_1 + m_2]$,

(H2) $p(\cdot)$ is monotone decreasing, and strictly monotone decreasing on the interval $\{q \in [0, m_1 + m_2]\,;\, p(q) > 0\}$,

(H3) $p(\cdot)$ is twice continuous differentiable at any $q = q_i + q_{-i}$ with $\partial\pi(q_i, q_{-i})/\partial q_i = 0$,

(H4) $\pi_i(q_i, q_{-i})$ is unimodal with respect to q_i,

(H5) If enterprise C_i produces the upper limit m_i for some $q_{-i} \in \overline{Q}_i$, then $m_i \in \varphi_i(q_{-i} - \varepsilon)$ for any $\varepsilon > 0$.

Note that, by (H4), mappings φ is single-valued.

Our main theorem is the following:

Theorem 1.1 (Main theorem) *Assume (H1)–(H5). Further assume that for each $i \in \{1, 2\}$ and for any $q_i \in (-\delta, m_i + \delta)$ $(\delta > 0)$ with $\partial\pi_i(q_i, q_{-i})/\partial q_i = 0$, one of the following three conditions is satisfied:*

(i) $p'' < 0$ *and* $q_i \neq \dfrac{-2p'}{p''}$,

(ii) $p'' = 0$,

(iii) $p'' > 0$ *and* $0 \le q_i \le \dfrac{-p'}{p''}$.

Then there exists a discrete Cournot-Nash equilibrium. In other words, there exists $q^* := (q_1^*, q_2^*) \in Q$ such that $q^* \in \phi(q^*)$.

Here we remark that the above theorem includes a classical situation where the price function is concave, see Examples 2.1 and 2.2 below.

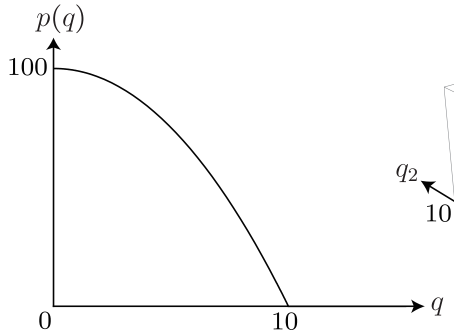
2 Examples

In this section, we give some examples of the price function that satisfy the assumption of the main theorem. We take $m_1 = m_2 = 10$ in the examples below.

Example 2.1 If we take

$$p(q) = \begin{cases} -q^2 + 100 & \text{if} \quad 0 \le q \le 10, \\ 0 & \text{if} \quad 10 < q, \end{cases}$$

and $c_1 = c_2 = 10$, then $p(\cdot)$ satisfies assumption (ii) and (H1)–(H5); see Figure 1 left. Figure 1 right is a graph of C_i's profit function. It is clear that $\pi_1(q_1, q_2)$ is unimodal with respect to q_1.



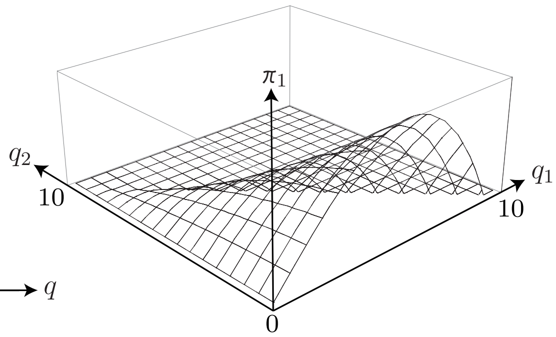
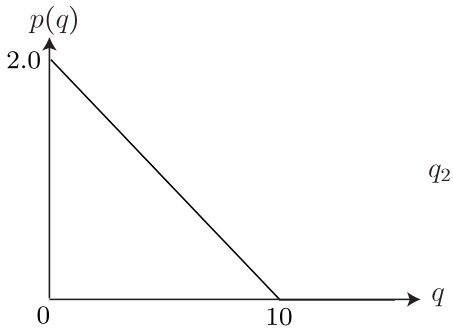


Fig. 1 Left: the price function, Right: C_1's profit function

Example 2.2 If we take

$$p(q) = \begin{cases} -\frac{1}{5}q + 2 & \text{if} \quad 0 \le q \le 10, \\ 0 & \text{if} \quad 10 < q, \end{cases}$$

and $c_1 = c_2 = 1$, then $p(\cdot)$ satisfies assumptions (i) and (H1)–(H5); see Figure 2 left.



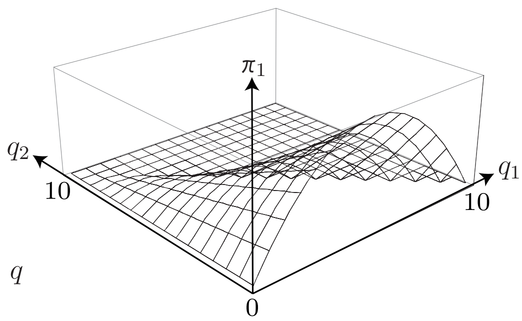


Fig. 2 Left: the price function, Right: C_1's profit function

Example 2.3 If we take

$$p(q) = \begin{cases} q^2 - 20q + 100 & \text{if} \quad 0 \leq q \leq 10, \\ 0 & \text{if} \quad 10 < q, \end{cases}$$

and $c_1 = c_2 = 10$, then $p(\cdot)$ satisfies assumptions (iii) and (H1)–(H5); see Figure 3 left.

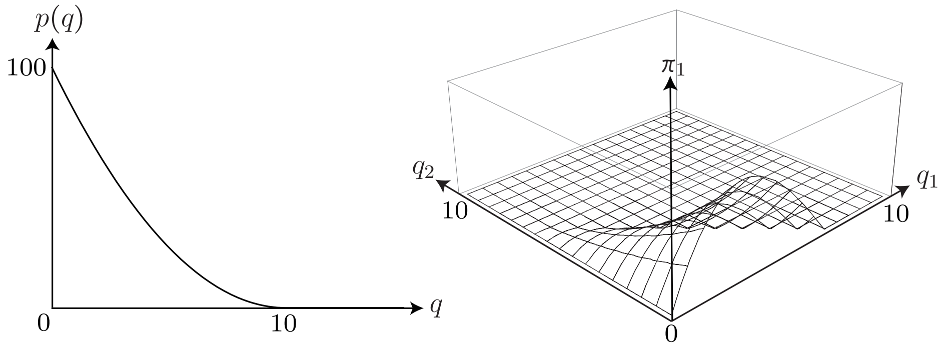


Fig. 3 Left: the price function, Right: C_1's profit function

Example 2.4 If we take $p(q) = -\arctan(q-10) + 2$ and $c_1 = c_2 = 2$, then $p(\cdot)$ satisfies assumptions (i)– (iii) and (H1)-(H5); see Figure 4 left.

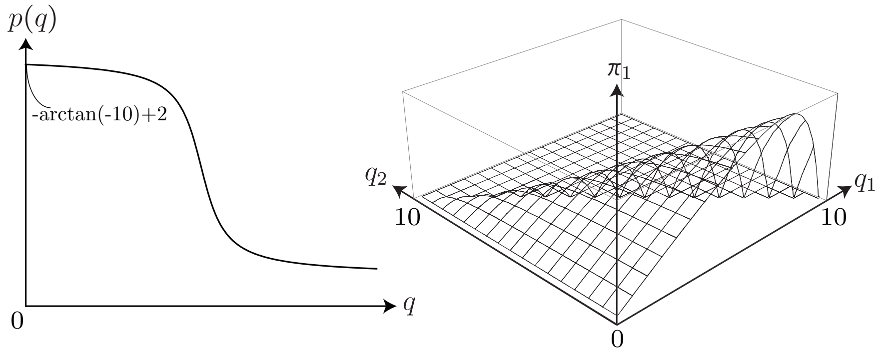


Fig. 4 Left: the price function, Right: C_1's profit function

3 Proof of Theorem 1.1

First, we quote a discrete fixed point theorem of [7], which is crucial in the proof of Theorem 1.1. Throughout this section, $V \subset \mathbb{Z}^n$, $(V, \leqq)$ is a partially ordered set and $f : V \to V$ is a nonempty set-valued mapping. Also, the symbol $x \preceq y$ means $x \leqq y$ and $x \neq y$.

Proposition 3.1 ([7, Theorem 2.2]) *Assume that there exists a sequence $\{x^k\}_{k \geq 0}$ in V such that $x^k \leqq x^{k+1} \in f(x^k)$ for any $k \geq 0$ and $\{x \in V;\ x^0 \leqq x\}$ is finite. Then, f has a fixed point $x^* \in f(x^*)$.*

In proving Theorem 1.1, we need the following two lemmas:

Lemma 3.1 *Assume that there exists $q^0_{-i} \in (-\delta, m_{-i} + \delta)$ such that $0 \in \varphi_i(q^0_{-i})$. Then we have $\varphi_i(q^0_{-i} + \varepsilon) = \{0\}$ for every positive ε.*

Proof. By $0 \in \varphi_i(q^0_{-i})$ and (H2), we get

$$p(0 + q^0_{-i} + \varepsilon) < p(0 + q^0_{-i}) \leq c_i,$$

that is, $p(0 + q^0_{-i} + \varepsilon) < c_i$. Therefore, C_i does not produce. Thus, it is holds that $\varphi_i(q^0_{-i} + \varepsilon) = \{0\}$. □

Lemma 3.2 *Assume that $\varphi_i(q_{-i}) \geq \varphi_i(q_{-i} + 1)$ for some $q_{-i} \in Q_{-i}$. Then we have $\phi_i(q_{-i}) \geq \phi_i(q_{-i} + 1)$.*

Proof. By the unimodality of π_i with respects to q_i, we have

$$\phi_i(q_{-i}) = \begin{cases} \{\lceil \varphi_i(q_{-i}) \rceil\} \\ \qquad \text{if} \quad \pi_i(\lceil \varphi(q_{-i}) \rceil, q_{-i}) > \pi_i(\lfloor \varphi(q_{-i}) \rfloor, q_{-i}), \\ \{\lceil \varphi_i(q_{-i}) \rceil, \lfloor \varphi_i(q_{-i}) \rfloor\} \\ \qquad \text{if} \quad \pi_i(\lceil \varphi(q_{-i}) \rceil, q_{-i}) = \pi_i(\lfloor \varphi(q_{-i}) \rfloor, q_{-i}), \\ \{\lfloor \varphi_i(q_{-i}) \rfloor\} \\ \qquad \text{if} \quad \pi_i(\lceil \varphi(q_{-i}) \rceil, q_{-i}) < \pi_i(\lfloor \varphi(q_{-i}) \rfloor, q_{-i}) \end{cases}$$

for any $q_{-i} \in Q_{-i}$, where $\lceil \cdot \rceil$ and $\lfloor \cdot \rfloor$ are rounding up and rounding down to the nearest integer, respectively. Thus, under the case where $\varphi_i(q_{-i}) - \varphi_i(q_{-i}+1) > 1$, we can immediately show $\phi_i(q_{-i}) \geq \phi_i(q_{-i} + 1)$.

Assume that $b+1 \geq \varphi_i(a) \geq \varphi_i(a+1) \geq b$ for some $a, b \in \mathbb{N} \cup \{0\}$. Further assume that $\phi_i(a) = \lceil \varphi_i(a) \rceil = b$ and $\phi_i(a+1) = \lfloor \varphi_i(a+1) \rfloor = b+1$. Since $\phi_i(a) = b$ is the best response, we have

$$\pi_i(\phi_i(a), a) \geq \pi_i(b+1, a). \tag{1}$$

Moreover, since $\phi_i(a+1) = b+1$ is the best response, we have

$$\pi_i(\phi_i(a+1), a+1) \geq \pi_i(b, a+1). \tag{2}$$

By (1) and (2), we easily see that

$$\Big(p(a+b+2) - p(a+b+1)\Big)(b+1) + \Big(p(a+b) - p(a+b+1)\Big)b \geq 0,$$

which contradicts assumption (H2). □

We are ready to prove Theorem 1.1

Proof of Theorem 1.1.

We define the following sets:

$$R^-_{-i} := \left\{ q_{-i} \in \overline{Q}_{-i}\,;\, p''(q) < 0,\ q_i \in \underset{r \in (-\delta, m_i+\delta)}{\operatorname{argmax}} \pi(r, q_{-i}) \right\},$$

$$R^0_{-i} := \left\{ q_{-i} \in \overline{Q}_{-i}\,;\, p''(q) = 0,\ q_i \in \underset{r \in (-\delta, m_i+\delta)}{\operatorname{argmax}} \pi(r, q_{-i}) \right\},$$

$$R^+_{-i} := \left\{ q_{-i} \in \overline{Q}_{-i}\,;\, p''(q) > 0,\ q_i \in \underset{r \in (-\delta, m_i+\delta)}{\operatorname{argmax}} \pi(r, q_{-i}) \right\}.$$

Here we note that the above sets can be empty.

For any $q_{-i} \in (-\delta, m_{-i}+\delta)$, if we take $q_i \in \varphi_i(q_{-i})$, then (I) $q_i \in (0, m_i)$ or (II) $q_i = 0$ or (III) $q_i = m_i$, because of $q_i \in [0, m_i]$.

(I) The case where $q_i \in (0, m_i)$: Production q_i is a solution of the following equation:

$$\frac{\partial \pi_i}{\partial q_i} = p'(q)q_i + p(q) - c_i = 0. \tag{3}$$

We put $h(q_i) := p'(q)q_i + p(q) - c_i$. Here we note that $h'(q_i) = p''(q)q_i + 2p'(q)$ and also $h'(\cdot) = \partial\pi_i^2/\partial^2 q_i$.

(I-i) The case where $q_{-i} \in R^-_{-i}$: By assumption (i), $p''(q)q_i + 2p'(q) \neq 0$, that is, $h'(q_i) \neq 0$. Therefore, by the implicit function theorem, (3) is uniquely solved on a neighborhood of (q_{-i}, c_i), say $U(q_{-i})$, such that $q_i = q_i(q_{-i})$. Namely,

$$p'(q_i(q_{-i}) + q_{-i})q_i + p(q_i(q_{-i})) - c_i = 0. \tag{4}$$

Differentiating (4) with respect to q_{-i}, we have

$$p''(q)\left(\frac{\partial q_i}{\partial q_{-i}}+1\right)q_i + p'(q)\frac{\partial q_i}{\partial q_{-i}} + p'(q)\left(\frac{\partial q_i}{\partial q_{-i}}+1\right) = 0,$$

which is equivalent to

$$\frac{\partial q_i}{\partial q_{-i}} = -\frac{p''(q)q_i + p'(q)}{p''(q)q_i + 2p'(q)}. \tag{5}$$

Since $p''(q)q_i + 2p'(q) < p''(q)q_i + p'(q) < 0$ is satisfied, it follows that

$$p''(q)q_i + p'(q) < 0 \quad \text{and} \quad p''(q)q_i + 2p'(q) < 0. \tag{6}$$

By (5) and (6), we have $\partial q_i/\partial q_{-i} < 0$ in $U(q_{-i})$ and $\partial^2\pi_i/\partial q_i^2 < 0$.

(I-ii) The case where $q_{-i} \in R^0_{-i}$: By assumptions (ii) and (H2), $2p'(q) < p'(q) < 0$ holds, and also $h'(q) \neq 0$ and $\partial^2\pi_i/\partial q_i^2 < 0$ hold. Thus, by similar discussion of the case where $q \in R^-_{-i}$, we have $\partial q_i/\partial q_{-i} < 0$ in $U(q_{-i})$.

(I-iii) The case where $q_{-i} \in R^+_{-i}$: By assumptions (iii) and (H2), it follows that $0 \geq q_i p''(q) + p'(q) > q_i p''(q) + 2p'(q)$. Thus, we have $h'(q) \neq 0$ and $\partial^2\pi_i/\partial q_i^2 < 0$. Therefore, by similar discussion of the case where $q \in R^-_{-i}$, we have $\partial q_i/\partial q_{-i} \leq 0$ in $U(q_{-i})$.

Since $\{U(q_{-i})\,;\, 0 \leq q_{-i} \leq m_{-i}\}$ is an open covering of $[0, m_{-i}]$, there exists a finite subcovering $\{U_j\}_{j=1}^m$, where $U_j := U_j(q_{-i})$. Also, by the uniqueness of the implicit function in neighborhood U_j, we get

$$\exists q_i = q_i(q_{-i}) \text{ s.t. } \frac{\partial q_i}{\partial q_{-i}} \leq 0 \text{ in } [0, m_{-i}].$$

Finally, since, by the unimodality of π_i with respects to q_i, local maximum implies global maximum, we get $q_i(q_{-i}) = \varphi_i(q_{-i})$.

(II) The case where $q_i = 0$: There exists $q^0_{-i} \in (-\delta, m_{-i} + \delta)$ such that $\varphi_i(q^0_{-i}) = 0$. Therefore, by Lemma 3.1, we get $\varphi_i(q_{-i}) = 0$ for all q_{-i} with $q^0_{-i} < q_{-i} \leq m_{-i}$.

(III) The case where $q_i = m_i$: There exists $q^M_{-i} \in (-\delta, m_{-i} + \delta)$ such that $\varphi_i(q^M_{-i}) = m_i$. Therefore, by (H5), we get $\varphi_i(q_{-i}) = m_i$ for all q_{-i} with $0 \leq q_{-i} < q^M_{-i}$.

By (I)–(III), $\varphi_i(q_{-i})$ is monotone decreasing with respect to q_{-i}.

Now we modify $\varphi_1(q_2)$ and $\varphi_2(q_1)$ so that they are integers, and define sequences $\{q_i^n\}_{n\geq 0} \subset Q_i$ $(i = 1, 2)$:

$$q_i^n := \begin{cases} \lceil \varphi_i(q_{-i}^n) \rceil \\ \qquad \text{if} \quad \pi_i(\lceil \varphi(q_{-i}^n) \rceil, q_{-i}^n) \geq \pi_i(\lfloor \varphi(q_{-i}^n) \rfloor, q_{-i}^n), \\ \lfloor \varphi_i(q_{-i}^n) \rfloor \\ \qquad \text{if} \quad \pi_i(\lceil \varphi(q_{-i}^n) \rceil, q_{-i}^n) < \pi_i(\lfloor \varphi(q_{-i}^n) \rfloor, q_{-i}^n). \end{cases}$$

Then the following holds:

- $q_1^n \in \phi_1(q_2^n)$ and $q_2^n \in \phi_2(q_1^n)$ for all $n \in \mathbb{Z}$,
- by Lemma 3.2, if $q_2^n < q_2^{n+1}$ then $q_1^n \geq q_1^{n+1}$, and if $q_1^n < q_1^{n+1}$ then $q_2^n \geq q_2^{n+1}$ for all $n \in \mathbb{Z}$.

We define the partial order $(q_1^1, q_2^1) \leqq (q_1^2, q_2^2)$ for $(q_1^1, q_2^1), (q_1^2, q_2^2) \in Q := Q_1 \times Q_2$ by $q_1^1 \leq q_1^2$ and $q_1^2 \geq q_2^2$. Then there exists a sequence $\{(q_1^n, q_2^n)\}_{n\geq 0} \subset Q$ satisfying $(q_1^n, q_2^n) \leqq (q_1^{n+1}, q_2^{n+1}) \in \phi(q^n)$ for all $n \geq 0$. Finally, applying Proposition 3.1 to $(V, f) = (Q, \phi)$, we conclude that ϕ has a discrete fixed point, which is a discrete Cournot-Nash equilibrium. □

4 Concluding remarks

It is not hard to calculate a discrete Cournot-Nash equilibrium for each example in Section 2. Indeed, $(3, 4)$, $(1, 2)$, $(2, 2)$ and $(3, 5)$ are a discrete Cournot-Nash equilibrium of Examples 2.1, 2.2, 2.3 and 2.4, respectively. Although Theorem 1.1 shows the existence of a discrete Cournot-Nash equilibrium, it does not mention the way to compute it, which is another important research theme.

By the way, there are several studies on discretized market competition models. We close this paper with introducing two types of these studies. One is based on discrete convex analysis proposed by Murota [6], and the other is based on discrete fixed point theorems.

Danilov-Koshevoy-Murota [3] is the first result of economic models based on discrete convex analysis. They considered the model called Arrow-Debreu type model. Furthermore, Danilov-Koshevoy-Lang [2] extended [3]'s model. Lehmann-Lehmann-Nisan [5] considered auctions. Tamura [8] expounded on these results.

On the other hand, Iimura [4] provided a discrete fixed point theorem and showed the existence of Walrasian equilibrium with indivisible commodities. Since his discrete fixed point theorem is based on Brouwer's fixed point theorem and the discrete convex analysis, it is a different type of fixed point theorem from Proposition 3.1.

Acknowledgments

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