Energy decay for a nonlinear generalized Klein-Gordon equation in exterior domains with a nonlinear localized dissipative term

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Abstract.

We give a certain energy decay rate for solutions of the exterior initial-boundary value problem of the nonlinear wave equations of the form:

$$u_{tt} - \Delta u + \rho(x, u_t) + g(u) = 0 \text{ in } \Omega \times R^+$$

$$u(x, 0) = u_0(x), u_t(x, 0) = u_1(x) \text{ and } u(x, t)|_{\partial\Omega} = 0$$

where Ω is an exterior domain in R^N, $\Omega = R^N/V$ with a compact set V, $\rho(x, v)$ is a function like $\rho(x, v) = a(x)|v|^r v, 0 \leq r \leq 2/(N-2)^+$, and $g(u)$ is a nonlinear term like $g(u) = k_0|u|^\alpha u, 0 < \alpha \leq 2/(N-2)^+, k_0 > 0$. $a(x)$ will be assumed to be positive near infinity and near a certain portion of the boundary.

We call our equation as a nonlinear generalized Klein-Gordon equation since the term $g(u)$ plays an essential role in our argument. Note that concerning energy decay, no result is known when both of $\rho(x, u_t)$ and $g(u)$ are nonlinear.