

Flat orbits, primary ideals and spectral synthesis

Abstract. Let $G = \exp \mathfrak{g}$ be a connected, simply connected, nilpotent Lie group and let ω be a continuous symmetric weight on G with polynomial growth. Let $l \in \mathfrak{g}^*$ be such that the co-adjoint orbit of l is flat. In the weighted group algebra $L_\omega^1(G)$ we characterize all the two-sided closed ideals whose hull is $\{\pi_l\}$, where π_l denotes the element of \hat{G} associated to the co-adjoint orbit $\text{Ad}^*(G)(l)$ by the Kirillov map. These ideals are parametrized by a set of G -invariant, translation invariant spaces of complex polynomials dominated by the weight ω and are realized as kernels of specially built induced representations. This results among others from the fact that, if the co-adjoint orbit of l is flat, every closed two-sided ideal of $L_\omega^1(G)$ with hull $\{\pi_l\}$ is necessarily $L^\infty(G/G(l))$ -invariant, where $G(l)$ denotes the stabilizer of l in G . (This is a joint work with J. Ludwig).