Sub-Laplacians of holomorphic L^p-type on exponential solvable and semi-simple Lie groups

W. Hebisch, J. Ludwig, D. Müller and S. Souaifi

Abstract

Let L denote a right-invariant sub-Laplacian on an G, endowed with a left-invariant Haar measure. Depending on the structure of G, and possibly also that of L, L may admit differentiable L^p-functional calculi, or may be of holomorphic L^p-type for a given $p \neq 2$. By "holomorphic L^p-type" we mean that every L^p-spectral multiplier for L is necessarily holomorphic in a complex neighborhood of some non-isolated point of the L^2-spectrum of L.

Assume that $p \neq 2$ and that G is an exponential, hence solvable Lie group . For a point ℓ in the dual $\mathfrak{g}^*$ of the Lie algebra $\mathfrak{g}$ of G, we denote by $\Omega(\ell) = Ad^*(G)\ell$ the corresponding coadjoint orbit. We prove that every sub-Laplacian on G is of holomorphic L^p-type, provided there exists a point $\ell \in \mathfrak{g}^*$ satisfying "Boidol's condition", such that the restriction of $\Omega(\ell)$ to the nilradical of $\mathfrak{g}$ is closed.

In contrast, if G is a semi-simple non compact Lie group with finite center, then every sublaplacian is of holomorphic L^p-type.