On a theorem of Wiener

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Abstract. Wiener has shown that an integrable function on the circle T which is square integrable near the identity and has nonnegative Fourier transform, is square integrable on all of T. In the last 30 years this has been extended by the work of various authors step by step. The latest result states that, in a suitable reformulation, Wiener's theorem with "p-integrable" in place of "square integrable" holds for all even p and fails for all other $p \in [1, \infty)$ in the case of a general locally compact abelian group. We extend this to all IN-groups and show that an extension to all locally compact groups is not possible: Wiener's theorem fails for all $p \in [1, \infty)$ in the case of the $ax + b$-group.