Transfer operators for $\Gamma_0(n)$ and the Hecke operators for the period functions of $\mathrm{PSL}(2,\mathbb{Z})$ [1]

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Abstract

In this article we report on a surprising relation between the transfer operators for the congruence subgroups $\Gamma_0(nm), n, m \in \mathbb{N}$, and some kind of Hecke operators on the space of vector valued period functions for the groups $\Gamma_0(n)$. For this we study special eigenfunctions of the transfer operators for the groups $\Gamma_0(nm)$ with eigenvalues ∓ 1 which are also solutions of the Lewis equations for these groups and which are determined by eigenfunctions of the transfer operator for the congruence subgroup $\Gamma_0(n)$. In the language of the Atkin-Lehner theory of old and new forms one should hence call them old eigenfunctions or old solutions of the Lewis equation for $\Gamma_0(n)$. It turns out that certain linear combinations of the components of these old solutions for the group $\Gamma_0(nm)$ determine for any m a solution of the Lewis equation for the group $\Gamma_0(n)$ and hence also an eigenfunction of the transfer operator for this group.

Our construction gives linear operators $\tilde{T}_n$ in the space of vector valued period functions for the group $\Gamma_0(n)$ which are rather similar to the Hecke operators. Indeed, in the case of the group $\Gamma_0(1) = \mathrm{SL}(2,\mathbb{Z})$ these operators are just the well known Hecke operators on the space of period functions for the modular group derived previously using the Eichler-Manin-Shimura correspondence between period polynomials and modular forms for this group and its extension to Maass wave forms by Lewis and Zagier.

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