Natural Parametrization for the Scaling Limit of Loop-Erased Random Walk in Three Dimensions

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joint work with Daisuke Shiraishi (Kyoto University)

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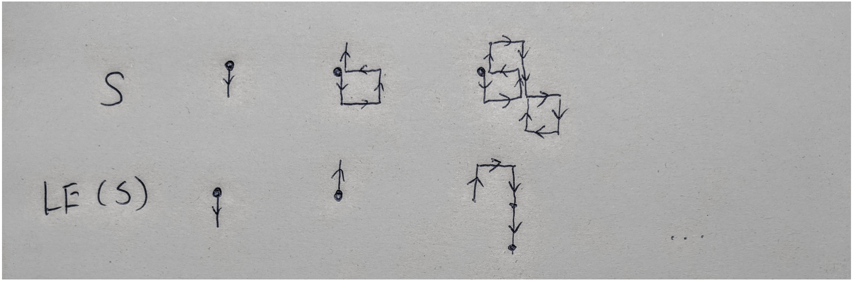
Chronological loop-erasure of a path

Loop-erased random walk (LERW) is the random simple path obtained by erasing all loops chronologically from a simple random walk path. In other words, we erase a loop immediately when it is created.



Precise definition of loop erasure (but don't look at it)

- Given a path $\lambda = [\lambda(0), \lambda(1), \cdots, \lambda(m)] \subset \mathbb{Z}^d$, we define its loop-erasure $\mathrm{LE}(\lambda)$ as follows. Let
$$s_0 := \max\{t \big| \lambda(t) = \lambda(0)\},$$
and for $i \geq 1$, let
$$s_i := \max\{t \big| \lambda(t) = \lambda(s_{i-1} + 1)\}.$$
We write $n = \min\{i \big| s_i = m\}$. Then we define $\mathrm{LE}(\lambda)$ by
$$\mathrm{LE}(\lambda) = [\lambda(s_0), \lambda(s_1), \cdots, \lambda(s_n)].$$

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- However, to save brainpower (and time!!), just imagine a kind of self-repulsive motion on the lattice.
- In fact, it is equivalent to a special case of Laplacian b−walk (again don't search for the definition on Google for the moment).

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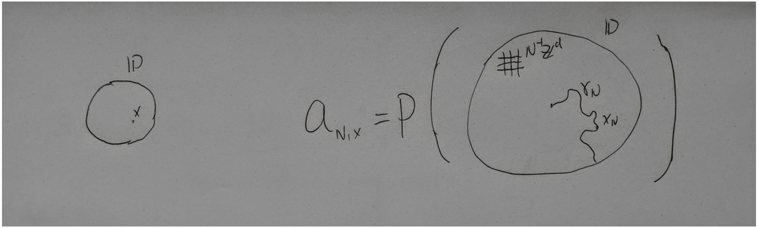

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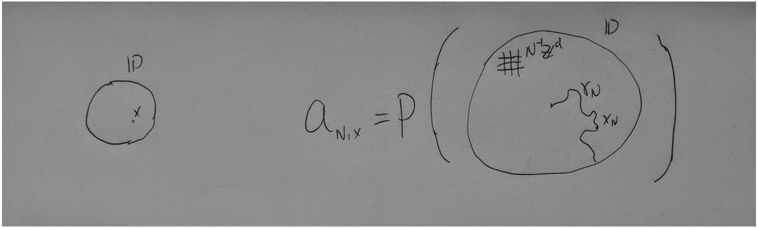

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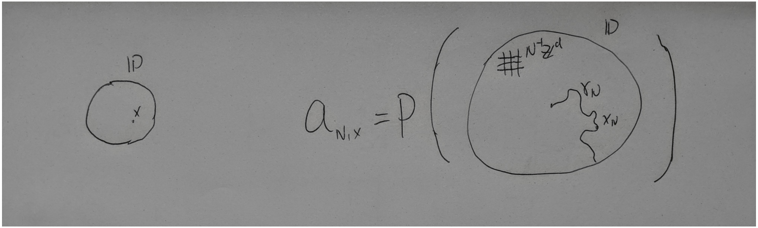

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- Let S_N be the simple random walk from 0 stopped at exiting D_N, and write $\gamma_N = \mathrm{LE}(S_N)$ for the LERW in D_N.
- For $x \in \mathbb{D}$, let x_N be its discretization. Let

$$a_{N,x} = P\Big(x_N \in \gamma_N\Big)$$

be the one-point function (or Green's function) of LERW.

Behaviour of LERW on $\mathbb{Z}^d$ in different dimensions

- LERW on $\mathbb{Z}^d$ enjoys a Gaussian behavior if d is large.
  - The scaling limit of LERW is the Brownian motion if $d \geq 4$.
  - Consider the one-point function $a_{N,x}$ for a fixed $x \in \mathbb{D}$. It is $O\big(N^{2-d}\big)$ for $d \geq 5$ and $O\big(N^{-2}(\log N)^{-1/3}\big)$ for $d = 4$.

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- When $d = 2$, the scaling limit of LERW is SLE_2[1] (Lawler-Schramm-Werner). Furthermore, we have the following asymptotics of the one-point function:

$$a_{N,x} = c_x N^{-3/4}\big(1 + O(N^{-c})\big). \text{ (Beneš-Lawler-Viklund)}$$

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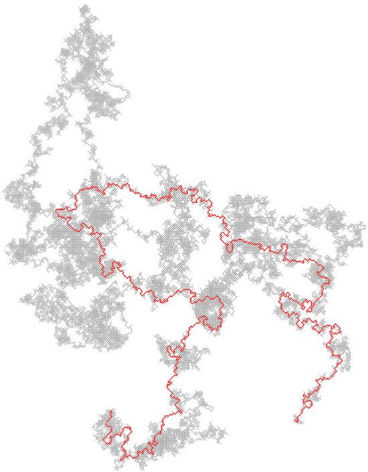
A simulation for 2D LERW



Picture credit: Fredrik Viklund.

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  - Numerical experiments and field-theoretical prediction suggest that $\beta = 1.62 \pm 0.01$, but there is no reason to believe that β is any nice number.

Results in 3D

Theorem (L.-Shiraishi '18)

There exist universal $\delta > 0$ *and* $c : \mathbb{D} \setminus \{0\} \to \mathbb{R}^+$ *such that* $\forall n \in \mathbb{Z}^+$ *and* $x \in \mathbb{D} \setminus \{0\}$,

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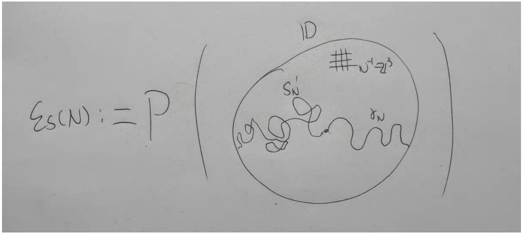
Remark

Our result also only works on dyadic scales, for relatively less fundamentally technical reasons. However, recently, we have found some tricks to extend the above theorem to any mesh size.

Non-intersection probability

- In parallel to the one-point function estimates, we also investigate the following non-intersection probability of simple random walk and LERW.



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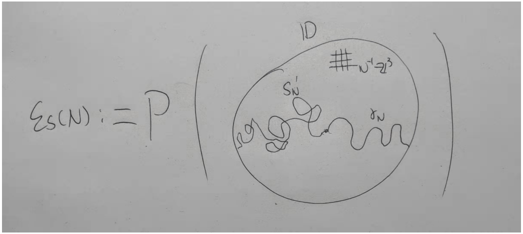

- Recall that S_N is the random walk from 0 stopped at exiting D_N and $\gamma_N = \mathrm{LE}(S_N)$. Let S'_N be an i.i.d. copy of S_N, ending at time T'. We are interested in

$$\mathrm{Es}(N) \triangleq P\big(\gamma_N \cap S'_N[1, T'] = \emptyset\big)$$

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- However, lattice effects around the origin and error bounds from Kozma's result do not allow us to do this directly.
- Solution: Replace the starting points by two points far away through conditioning and modifying a recent coupling result from Greg Lawler (*The infinite two-sided loop-erased random walk*, arXiv:1802.06667).

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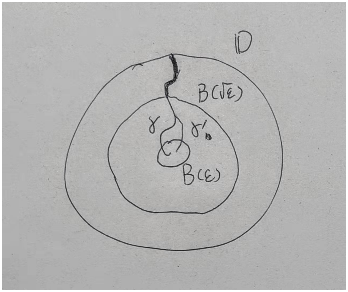

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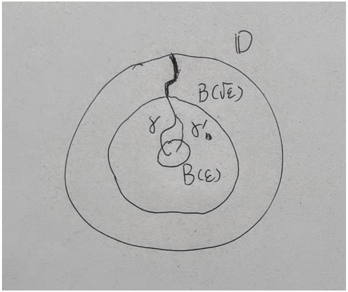
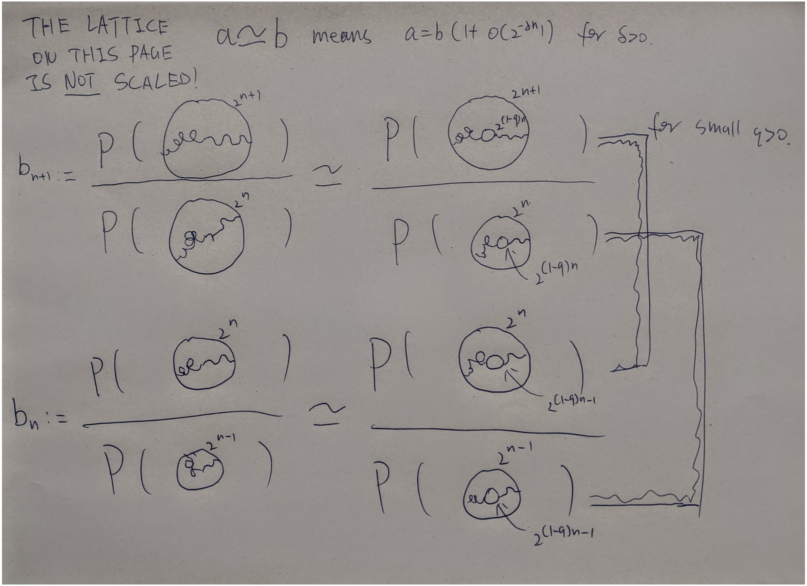


- Then it is possible to couple γ and γ' such that their "outer" part agree with high probability.
- This allows us to separate the "local" behavior and the "global" behaviour of LERW.

Let $b_n = \mathrm{Es}(2^n)/\mathrm{Es}(2^{n-1})$. Want to show $b_{n+1}/b_n = 1 + O(2^{-\delta n})$.



Convergence of 3D LERW to its scaling limit in natural parametrization

- Recall that D_N is the discretized unit ball with mesh size $1/N$, and let γ_N is the LERW on $N^{-1}\mathbb{Z}^3$, from the origin and stopped at exiting D_N. Recall that $\beta \in (1, 5/3]$ is the Hausdorff dimension of $\mathcal{K}$.

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in the product topology of $(\mathcal{H}(\overline{\mathbb{D}}); d_{\mathrm{Haus}})$ and the topology of the weak convergence on $\mathcal{M}(\overline{\mathbb{D}})$. Moreover, μ^* is measurable w.r.t. $\mathcal{K}$.

Here

- $\mathcal{H}(\overline{\mathbb{D}})$ is the space of non-empty compact subsets of $\overline{\mathbb{D}}$;
- d_{Haus} is the the Hausdorff metric on $\mathcal{H}(\overline{\mathbb{D}})$;
- $\mathcal{M}(\overline{\mathbb{D}})$ is the space of finite measures on $\overline{\mathbb{D}}$.

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    $d_\infty(\lambda_1, \lambda_2) = \max_{0 \le s \le 1} \left| \lambda_1(st_{\lambda_1}) - \lambda_2(st_{\lambda_2}) \right| + \left| t_{\lambda_1} - t_{\lambda_2} \right|;$
  - $\eta_N(t) := \gamma_N(N^\beta t)$ be the properly time-rescaled LERW as an element of $\mathcal{C}(\overline{\mathbb{D}})$;

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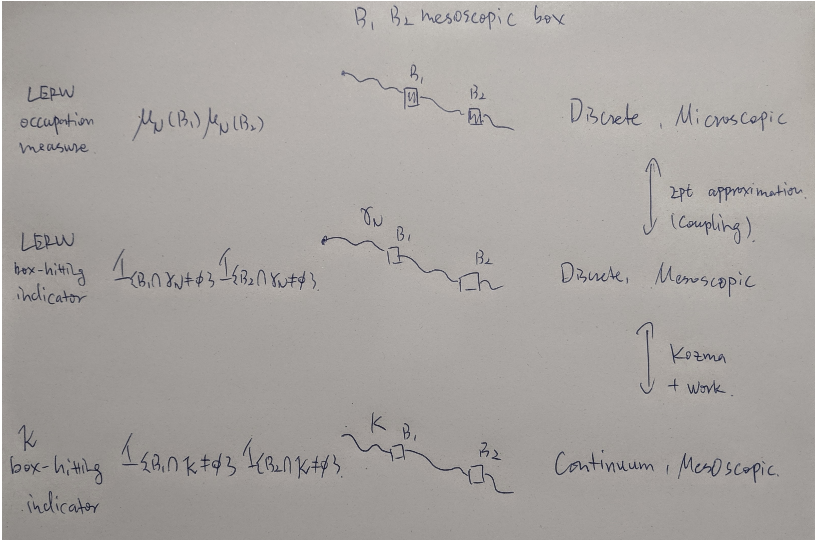
Theorem (L.-Shiraishi '18)

As $n \to \infty$, $\eta_{2^n} \stackrel{\mathrm{w}}{\to} \eta^*$ with respect to the topology of $(\mathcal{C}(\overline{\mathbb{D}}), d_\infty)$.

Remark

As γ_{2^n} is traversed at a constant speed, what we obtain is a convergence in the natural parametrization. We conjecture that μ^* can also be given through the Minkowski content of $\mathcal{K}$.

L^2-approximation in the style of Garban-Pete-Schramm



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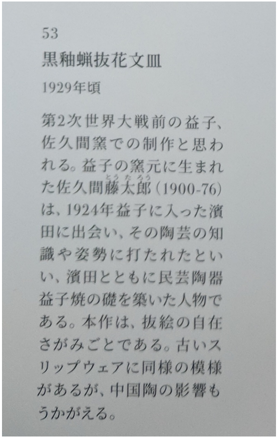
  Preprint, available at arXiv:1807.00541, 39 pages, 2 figures.

- X. Li and D. Shiraishi.
  Natural parametrization for the scaling limit of loop-erased random walk in three dimensions. *Preprint*, available at arXiv:1811.11685, 74 pages, 3 figures.

Thank you for your attention!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黒釉蝋抜花文皿

1929年頃

第2次世界大戦前の益子、佐久間窯での制作と思われる。益子の窯元に生まれた佐久間藤太郎(1900-76)は、1924年益子に入った濱田に出会い、その陶芸の知識や姿勢に打たれたといい、濱田とともに民芸陶器益子焼の礎を築いた人物である。本作は、抜絵の自在さがみごとである。古いスリップウェアに同様の模様があるが、中国陶の影響もうかがえる。

-Mashikoyaki pottery made by Shoji HAMADA et al., ca. 1929.