Large deviations on the profile of mutation effects

for a model of populations adapting to a changing environment

Given a function $F : \mathcal{X} \times \mathcal{X}$ vanishing on the diagonal,
we want to study large deviations of $\langle J_t \,|\, F \rangle := (1/t) \times \sum_{s \leq t} F(X_{s-}, X_s)$
conditionally upon $\{t < \tau_\partial\}$, for large t :

For any $\gamma \in Im(\psi_F)^\circ$, for any $x \in \mathcal{X}$

$$\lim_{t\to\infty} \frac{1}{t} \log \mathbb{P}_x \left[\langle J_t \,|\, F \rangle \geq \gamma \,\middle|\, t < \tau_\partial \right] = \lambda_0^{(c_\gamma F)} - \lambda_0^{(0)} - \gamma\, c_\gamma,$$

where $c_\gamma = \psi_F^{-1}(\gamma)$.

Here $\psi_F(c)$ and $\lambda_0^{(cF)}$ are given by the quasi-ergodicity of the process X biased by some Feynman-Kac penalization of the form : $V_t := c\, t \,\langle J_t \,|\, F \rangle = \sum_{s \leq t} F(X_{s-}, X_s)$.

Velleret Aurélien, Institut de Mathématique de Marseille, AMU

Feynman-Kac quasi-ergodicity

$$\lambda_0^{(cF)} = \lim_{t\to\infty} \frac{1}{t} \log \mathbb{E}_x \left[\exp(ct \left\langle J_t \,\middle|\, F \right\rangle) \,,\; t < \tau_\partial\right].$$

$\psi_F(c)$ is the value for which for any μ and $\epsilon > 0$:

$$\lim_{t\to\infty} \mathbb{Q}_\mu^{(cF),t} \left[\left|\left\langle J_t \,\middle|\, F\right\rangle - \psi_F(c)\right| > \epsilon\right] = 0,$$

where for any $\mathcal{F}_t$ measurable Λ_t :

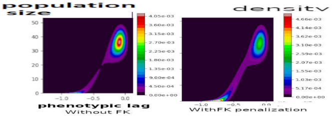
$$\mathbb{Q}_\mu^{(cF),t}[\Lambda_t] = \frac{\mathbb{E}_\mu\left[\exp[ct \left\langle J_t \,\middle|\, F\right\rangle] \;;\; \Lambda_t, t < \tau_\partial\right]}{\mathbb{E}_\mu\left[\exp[ct \left\langle J_t \,\middle|\, F\right\rangle] \;;\; t < \tau_\partial\right]}$$
$$= \frac{\mathbb{E}_\mu\left[\exp[c \sum_{s\le t} F(X_{s-}, X_s)] \;;\; \Lambda_t, t < \tau_\partial\right]}{\mathbb{E}_\mu\left[\exp[c \sum_{s\le t} F(X_{s-}, X_s)] \;;\; t < \tau_\partial\right]}.$$

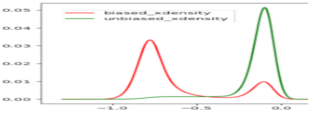
Velleret Aurélien, Institut de Mathématique de Marseille, AMU

First hints of simulations

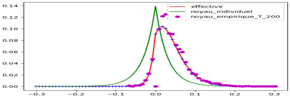
$\exp(F(x,y)) = 0.8$ iff $y - x \in (0, 0.1)$

We see clearly the destabilization of the system, with a much larger extinction rate.

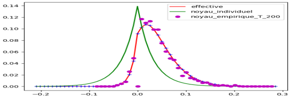




On the other hand, the profile of mutations seems almost unchanged. So it seems that this direction of deformation is particularly costly.



without penalization



with penalization