The differentiability of the speed of biased RWs on Galton-Watson trees

Yuki Tokushige(RIMS, Kyoto University)
joint with A. Bowditch (National University of Singapore)

Setting

- $\mathbf{T}$:= a supercritical Galton-Watson tree with offspring distribution $\{p_k\}_{k\geq 0}$. Define the generating function $f(s) := \sum_{k\geq 0} p_k s^k$.
- Consider **λ-biased RWs** $(Z^\lambda(n))$ on $\mathbf{T}$. $(\lambda > 0)$

Lyons-Pemantle-Peres(1995,96)

Let $q \in [0,1)$ be the extinction probability.

1. The limit $\lim_{n\to\infty} n^{-1} \cdot distance(\mathrm{root}, Z^\lambda(n))$ exists **Annealed**(λ)-a.s., and the limit $v(\lambda)$ is a **deterministic** constant.
2. When $\lambda \geq \mu$ or $0 < \lambda \leq f'(q)$, we have $v(\lambda) = 0$.
3. When $f'(q) < \lambda < \mu$, we have $v(\lambda) > 0$.

<u>**AIM:**</u> To analyze the **function** $\lambda \mapsto v(\lambda)$.

Result

Bowditch-T.(2019+)

Assume that there exists $\beta > 1$ such that $\sum_{k\geq 0} p_k \beta^k < +\infty$.

- When $p_0 = 0$, the function $\lambda \mapsto v(\lambda)$ is **differentiable** in $(0, \mu)$.
- When $p_0 > 0$, the function $\lambda \mapsto v(\lambda)$ is **differentiable** in $(\sqrt{f'(q)}, \mu)$.

In either case, the derivative $\frac{dv(\lambda)}{d\lambda}$ has an expression using **Gaussian random variables**.

Remark

1. *Berger-Gantert-Nagel(2019) proved a very similar result for biased RWs on $\mathbb{Z}^d$ with i.i.d. uniformly elliptic weights.*
2. *Aidekon(2013) proved the differentiability of $\lambda \mapsto v_\lambda$ in $(0, 1)$ when $p_0 = 0$.*