Isomorphisms between DPPs and PPPs

$\hat{K} \in L^1(\mathbb{R}^d)$ be a $[0,1]$-valued function.

$$K(x,y) = \int_{\mathbb{R}^d} \hat{K}(t) \mathrm{e}^{2\pi \mathrm{i}(x-y)\cdot t} dt.$$

THM

K-determinantal point process is isomorphic to Poisson point processes.

- Poisson point processes are isomorphic to each other regardless of their intensities.
- We cannot distinguish DPPs as measure preserving dynamical system.

Isomorphisms between DPPs and PPPs

- $(\Omega, \mathcal{F}, \mathbb{P}, \mathsf{T}_G)$: <u>$G$-action system</u>
- $(\Omega, \mathcal{F}, \mathbb{P}, \mathsf{T}_G)$ and $(\Omega', \mathcal{F}', \mathbb{P}', \mathsf{T}'_G)$ are <u>isomorphic</u>
  $\overset{def}{\Leftrightarrow}$ $^\exists\phi : \Omega_0 \to \Omega'_0$: bi-measurable bijection s.t.

$$\begin{cases} \mathbb{P}(\Omega_0) = \mathbb{P}'(\Omega'_0) = 1 \\ \mathbb{P} \circ \phi^{-1} = \mathbb{P}' \\ \phi \circ \mathsf{T}_g = \mathsf{T}'_g \circ \phi \text{ for } {}^\forall g \in G. \end{cases}$$

ex. $(d = 1)$ Sine$_2$ point process

$$\frac{\sin \pi(x-y)}{\pi(x-y)} = \int_{\mathbb{R}} 1_{\{|t| \le \pi\}}(t) \mathrm{e}^{2\pi \mathrm{i}(x-y)\cdot t} dt$$