Random Weighted Shifts

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Motivation

"Random Matrix Theory" vs. "Random Operator Theory"

- Little is known for non-selfadjoint and bounded ROT.
- We start with a canonical example: the unilateral shift

H: a Hilbert space with orthonormal basis: $e_0, e_1, \cdots, e_n, \cdots$
The unilateral shift: $Te_n = e_{n+1}$ or $T(\sum c_n e_n) = \sum c_n e_{n+1}$.

We introduce the random version of the unilateral shift:

$$Te_n = X_n e_{n+1}.$$

$\{X_1, X_2, \cdots, X_n, \cdots\}$: i.i.d. positive random variables.

Two Theorems

Theorem A: If X_1 is non-constant and $||X_1||_\infty = 1$, then for any $\|A\| < 1$, there exist a.s. $H = H_1 \oplus H_2 \oplus H_3$ and

$$T \cong \begin{pmatrix} * & * & * \\ 0 & A & * \\ 0 & 0 & * \end{pmatrix}$$

Theorem B: There exists an associated random Hardy space

$$H^2_\mu = \left\{ f(z) = \sum_{n=0}^{\infty} a_n z^n : \ \|f\|^2 = \sum_{n=0}^{\infty} |a_n|^2 w_n^2 < \infty \right\}$$

Then $P(\|f\| < \infty) \in \{0, 1\}$ for any f. Define $H_* =$ the good set.

If X_1 is non-constant and $E(\ln X_1) = 0$, then

$$\sup_{f \in H_*} \limsup_{n \to \infty} |a_n|^{\frac{\sqrt{2}}{\sigma \sqrt{n \ln \ln n}}} \in [e^{-1}, 1].$$