

**The 12th MSJ-SI**  
The Mathematical Society of Japan, Seasonal Institute  
**Stochastic Analysis,**  
**Random Fields and Integrable Probability**

**Program and Abstracts**

July 31 (Wed.) – August 9 (Fri.), 2019  
Lecture Theater 5, Shiiki Hall, Ito Campus, Kyushu University

This symposium is sponsored by the Mathematical Society of Japan (MSJ) and supported by Faculty of Mathematics, Institute of Mathematics for Industry and Stochastic Analysis Research Center at Kyushu University, as well as by Japan Society for the Promotion of Science, Grant-in-Aid for:

Scientific Research (S) 16H06338 PI: H. Osada (Kyushu University);  
Scientific Research (A) 17H01093 PI: T. Kumagai (Kyoto University);  
Scientific Research (A) 19H00643 PI: M. Hino (Kyoto University).

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## Timetable & Program (1st Week)

July 31	August 1	August 2
10:00-11:00 den Hollander	10:00-11:00 Corwin	10:00-11:00 den Hollander
11:20-12:20 Corwin	11:20-12:20 Corwin	11:20-12:20 den Hollander
Lunch		
13:40-14:20 Funaki	13:40-14:20 Imamura	13:40-14:20 Rodriguez
14:30-15:10 Zhu	14:30-15:10 Holden	14:30-15:10 Berestycki
15:20-15:40 Hoshino	15:20-15:40 Abe	15:20-15:40 Okada
Coffee Break		
16:10-16:50 Ghosh	16:10-16:50 Katori	16:10-16:50 Tsai
17:00-17:20 Tsunoda	17:00-17:20 Małeckı	17:00-17:20 Li
18:00-20:00 Welcome Party		

### July 31 (Wed.)

- 9:00- Registration
- 9:45- Opening Speech  
Tomohide Terasoma (President of MSJ)
- 10:00-11:00 Frank den Hollander (Universiteit Leiden)  
Complex Networks: Structure and Functionality. I. Spectra
- 11:20-12:20 Ivan Corwin (Columbia University)  
Stochastic Vertex Models I
- 13:40-14:20 Tadahisa Funaki (Waseda University)  
Large Deviation for Lozenge Tiling Dynamics
- 14:30-15:10 Rongchan Zhu (Beijing Institute of Technology)  
Stochastic Cahn-Hilliard Equation
- 15:20-15:40 Masato Hoshino (Kyushu University)  
Paracontrolled Calculus and Regularity Structures
- 16:10-16:50 Subhroshekhar Ghosh (National University of Singapore)  
Two Manifestations of Rigidity in Point Sets: Forbidden Regions and  
Maximal Degeneracy
- 17:00-17:20 Kenkichi Tsunoda (Osaka University)  
Derivation of Viscous Burgers Equations from Weakly Asymmetric  
Exclusion Processes
- 17:20- *Group Photo*

18:00-20:00 Welcome Party (Big Orange)

**August 1 (Thu.)**

10:00-11:00 Ivan Corwin (Columbia University)  
Stochastic Vertex Models II

11:20-12:20 Ivan Corwin (Columbia University)  
Stochastic Vertex Models III

13:40-14:20 Takashi Imamura (Chiba University)  
Determinantal Structures in the  $q$ -Whittaker Measure

14:30-15:10 Nina Holden (ETH-ITS Zurich)  
Cardy Embedding of Random Planar Maps

15:20-15:40 Yoshihiro Abe (Chiba University)  
Exceptional Points of Two-Dimensional Random Walks at Multiples of the  
Cover Time

16:10-16:50 Makoto Katori (Chuo University)  
Gaussian Free Fields with Boundary Points, Multiple SLEs, and Log-Gases

17:00-17:20 Jacek Malecki (Wroclaw University of Science and Technology)  
Universality Classes for General Random Matrix Flows

**August 2 (Fri.)**

10:00-11:00 Frank den Hollander (Universiteit Leiden)  
Complex Networks: Structure and Functionality. II. Equivalence

11:20-12:20 Frank den Hollander (Universiteit Leiden)  
Complex Networks: Structure and Functionality. III. Exploration

13:40-14:20 Pierre-François Rodriguez (Institut des Hautes Études  
Scientifiques)  
Isomorphism Theorems and the Sign Cluster Geometry of the Gaussian  
Free Field

14:30-15:10 Julien Berestycki (The University of Oxford)  
Branching Particle Systems with Selection and Free Boundary Problems

15:20-15:40 Izumi Okada (Kyushu University)  
Exponents for High Points of Simple Random Walks in Two Dimensions

16:10-16:50 Li-Cheng Tsai (Rutgers University)  
Lower-Tail Large Deviations of the KPZ Equation

17:00-17:20 Xinyi Li (The University of Chicago)  
Natural Parametrization for the Scaling Limit of Loop-Erased Random  
Walk in Three Dimensions

## Timetable & Program (2nd Week)

August 5	August 6	August 7	August 8	August 9
10:00-11:00 Lawler	10:00-11:00 Addario-Berry	10:00-11:00 Miermont	10:00-11:00 Bufetov	10:00-11:00 Bufetov
11:20-12:20 Addario-Berry	11:20-12:20 Lawler	11:20-12:20 Addario-Berry	11:20-12:20 Miermont	11:20-12:20 Bufetov
Lunch		Excursion	Lunch	
13:40-14:40 Lawler	13:40-14:20 Sasada		13:40-14:40 Miermont	13:40-14:20 Kajino
14:50-15:30 Shiraishi	14:30-15:10 Basu		14:50-15:30 Sakai	14:30-15:10 Fukushima
15:40- Poster Presentations	15:20-15:40 Nakajima		Coffee Break	
	Coffee Break		16:00-16:40 Kusuoka	
16:40- Poster Session	16:10-16:50 Seo		16:50-17:30 Qiu	
	18:30-20:30 Banquet			

### August 5 (Mon.)

10:00-11:00 Gregory F. Lawler (The University of Chicago)

Loop Measures and Loop-Erased Random Walk I

11:20-12:20 Louigi Addario-Berry (McGill University)

Algorithms and Random Trees I

12:20- *Group Photo*

13:40-14:40 Gregory F. Lawler (The University of Chicago)

Loop Measures and Loop-Erased Random Walk II

14:50-15:30 Daisuke Shiraishi (Kyoto University)

Scaling Limit of Uniform Spanning Tree in Three Dimensions

15:40- Poster (Short Presentations Followed by the Poster Session)

### August 6 (Tue.)

10:00-11:00 Louigi Addario-Berry (McGill University)

Algorithms and Random Trees II

11:20-12:20 Gregory F. Lawler (The University of Chicago)

Loop Measures and Loop-Erased Random Walk III

13:40-14:20 Makiko Sasada (The University of Tokyo)

Generalized Pitman's Transform and Discrete Integrable Systems

14:30-15:10 Riddhipratim Basu (Tata Institute of Fundamental Research)

Bigeodesics and Polymer Watermelons in Last Passage Percolation

15:20-15:40 Shuta Nakajima (Nagoya University)

Divergence of Non-Random Fluctuation in First-Passage Percolation

16:10-16:50 Insuk Seo (Seoul National University)  
The Cut-Off Phenomenon for the Random-Cluster Model

18:30-20:30 Banquet (Hyakunen-Gura)

### **August 7 (Wed.)**

10:00-11:00 Grégory Miermont (École Normale Supérieure de Lyon)  
Brownian Surfaces I

11:20-12:20 Louigi Addario-Berry (McGill University)  
Algorithms and Random Trees III

Excursion (Karatsu)

### **August 8 (Thu.)**

10:00-11:00 Alexander I. Bufetov (Aix-Marseille Université)  
Determinantal Point Processes and Extrapolation I

11:20-12:20 Grégory Miermont (École Normale Supérieure de Lyon)  
Brownian Surfaces II

13:40-14:40 Grégory Miermont (École Normale Supérieure de Lyon)  
Brownian Surfaces III

14:50-15:30 Akira Sakai (Hokkaido University)  
Critical Two-Point Function for Long-Range Models with Power-Law  
Couplings: The Marginal Case for  $d \geq d_c$

16:00-16:40 Seiichiro Kusuoka (Kyoto University)  
Approach to the Quantum Field with Exponential Interactions by Singular  
SPDEs

16:50-17:30 Yanqi Qiu (Chinese Academy of Sciences)  
A Class of Positive Functions on the Circle and Their Applications on  
Hankel Operators

### **August 9 (Fri.)**

10:00-11:00 Alexander I. Bufetov (Aix-Marseille Université)  
Determinantal Point Processes and Extrapolation II

11:20-12:20 Alexander I. Bufetov (Aix-Marseille Université)  
Determinantal Point Processes and Extrapolation III

13:40-14:20 Naotaka Kajino (Kobe University)  
The Laplacian on Some Self-Conformal Fractals and Weyl's Asymptotics  
for Its Eigenvalues

14:30-15:10 Ryoki Fukushima (Kyoto University)  
Zero Temperature Limits for Directed Polymers in Random Environment

15:15- Closing Speech  
Hirofumi Osada (Chair of the Scientific Committee)

# Serial Lectures

## Algorithms and Random Trees

Louigi Addario-Berry (McGill University)

I will present a variety of recent and ongoing work at the interface of probabilistic analysis of algorithms and the study of random trees.

First, I will discuss an ongoing project on how certain recursive distributional equations can be solved by importing rigorous results on the convergence of approximation schemes for degenerate PDEs, from numerical analysis.

Second, I will discuss joint work with Pascal Maillard on the algorithmic hardness of finding low-energy states in the continuous random energy model of Bovier and Kurkova.

Finally, I will also discuss one or both of (a), (b) and (c), where (a) is recently completed work with Marie Albenque on symmetrization of random trees and its application to convergence of random maps, (b) is an ongoing project about eigenvalues of random trees, and (c) is joint work with Sanchayan Sen, on distributional convergence of the minimum spanning tree of random 3-regular graphs.



## Determinantal Point Processes and Extrapolation

Alexander I. Bufetov (Aix-Marseille Université)

Point processes whose correlation functions are given by determinants arise in the study of very different objects such as, for example, random matrices (Dyson), fermions (Macchi), random series (Peres-Virag, Krishnapur), spanning trees (Burton-Pemantle, Benjamini-Lyons-Peres-Schramm), Young diagrams (Baik-Deift-Johansson, Borodin-Okounkov-Olshanski), non-intersecting diffusions (Johansson, Osada-Tanemura), representations of infinite-dimensional groups (Borodin-Olshanski).

For general determinantal point processes a rich general theory has been built, which includes existence theorems (Macchi, Soshnikov, Shirai-Takahashi), description of Palm measures (Shirai-Takahashi, Lyons), rigidity (Ghosh-Peres), the Central Limit Theorem (Soshnikov) and its functional analogue (joint work with Dymov).

The course will start with an elementary introduction to determinantal point processes, proceeding to recent developments and open problems.

For example, consider a Gaussian Analytic Function on the disk, that is, a random series whose coefficients are independent complex Gaussians. In joint work arXiv:1612.06751 with Yanqi Qiu and Alexander Shamov, we show that the zero set of a Gaussian Analytic Function is a uniqueness set for the Bergman space on the disk: in other words, almost surely, there does not exist a nonzero square-integrable holomorphic function having these zeros. The key rôle in our argument is played by the determinantal structure of the zeros due to Peres and Viràg. In general, we prove that the family of reproducing kernels along a realization of a determinantal point process generates the whole ambient Hilbert space, thus settling a conjecture of Lyons and Peres.

The key lemma in our argument is that the determinantal property is preserved under conditioning on a fixed restriction of our configuration in a part of the phase space. In full generality, the explicit description of the kernels of these conditional measures remains an open problem.

It is nevertheless possible to write the analogue of the Gibbs property for one-dimensional determinantal point processes with integrable kernels: for these processes, conditional measures with respect to fixing the configuration in the complement of an interval is an orthogonal polynomial ensemble with explicitly found weight (arXiv:1605.01400). Note here that a reproducing kernel must have integrable form as soon as the corresponding Hilbert space satisfies a weak form of division property (joint work with Roman Romanov); in this case, our space must be a Hilbert space of holomorphic functions.

The proof of the Lyons-Peres conjecture raises the problem of extrapolating a function from its restriction to a realization of a determinantal point process. In joint work with Yanqi Qiu (arXiv:1806.02306), extrapolation for Bergman functions from zero sets of Gaussian Analytic Functions is obtained using the Patterson-Sullivan construction.

## Stochastic Vertex Models

Ivan Corwin (Columbia University)

We will describe some recent advances in the study of stochastic vertex models. Topics may include the Yang Baxter equation and Bethe ansatz, Markov dualities, and asymptotics to various stochastic PDEs and the KPZ fixed point. No prior knowledge of these subjects will be assumed.

### Complex Networks: Structure and Functionality.

#### I. Spectra; II. Equivalence; III. Exploration

Frank den Hollander (Universiteit Leiden)

Lecture I considers inhomogeneous *Erdős-Rényi random graphs* on  $n$  vertices. We study the *empirical spectral distribution* of the *adjacency matrix*  $A_n$  in the limit as  $n \rightarrow \infty$  in a regime that interpolates between sparse and dense. In particular, we show that the empirical spectral distribution of  $A_n$  when properly scaled converges to a deterministic limit weakly in probability. For the special case where the connectivity probability between two vertices has the *product property*, we give an explicit characterisation of the limit distribution. The result is applied to statistical inference of *sociability patterns in social networks*. (Based on joint work with Arijit Chakrabarty, Rajat Hazra and Matteo Sfragara.)

Lecture II considers random graphs subject to topological constraints. We compare two probability distributions on the set of simple graphs on  $n$  vertices induced by a given constraint: (1) The *microcanonical ensemble*, where the constraint is *hard*, i.e., has to be satisfied for every realisation of the graph; (2) The *canonical ensemble*, where the constraint is *soft*, i.e., has to be satisfied on average. We say that *breaking of ensemble equivalence* occurs in the limit as  $n \rightarrow \infty$  when the *relative entropy* of the two ensembles per vertex (in the sparse regime), respectively, per edge (in the dense regime) is strictly positive. We present two examples of constraints where breaking of ensemble equivalence occurs, namely, when the constraint is on the degree sequence and when the constraint is on the total number of edges and triangles. The result is applied to model selection for real-world networks. (Based on joint work with Diego Garlaschelli, Michel Mandjes, Andrea Roccaverde, Tiziano Squartini and Nicos Starreveld.)

Lecture III considers the mixing time of random walks on random graphs. Many real-world networks, such as WWW, are dynamic in nature. It is therefore natural to study random walks on *dynamic random graphs*. We consider random walk on a random graph with prescribed degrees. We investigate what happens when at each unit of time a fraction  $\alpha_n$  of the edges is randomly rewired, where  $n$  is the number of vertices. We identify *three regimes* for the mixing time in the limit as  $n \rightarrow \infty$ , depending on the choice of  $\alpha_n$ . These regimes exhibit surprising behaviour. The results are relevant for Google PageRank. (Based on joint work with Luca Avena, Hakan Guldas and Remco van der Hofstad.)

## Loop Measures and Loop-Erased Random Walk

Gregory F. Lawler (The University of Chicago)

I will give an introduction to (discrete time, discrete space) loop measures and the loop-erased random walk (LERW), along with the closely related objects, uniform spanning trees and the Gaussian free field. The first lecture will emphasize the discrete loop measures and will be combinatorial in nature; the second will discuss the LERW in dimensions 2 through 4; and the third will discuss conformal invariance and the scaling limits in two dimensions (Schramm-Loewner evolution, Brownian loop measure).

## Brownian Surfaces

Grégory Miermont (École Normale Supérieure de Lyon)

I will review the topic of random maps and its connection to classical objects of probability theory, namely random walks and peeling processes, and branching processes arising in slice decompositions and layer decompositions.

The first lecture will focus on the “local” model of Boltzmann maps and will comment on its relation to “non-local” models of maps, and will introduce peeling processes and some applications.

The second and third lectures will focus on slice decompositions, which is, from the geometric point of view, a convenient way to understand the Bouttier-di Francesco-Guitter bijection between maps and trees. We will show how this decomposition can be used to show convergence of random maps to Brownian surfaces in various topologies.

Time permitting, I will comment on the use by Curien-Le Gall of layer decompositions to show robustness of these results when modifying locally the distances, and how it is used by Carrance to show convergence of Eulerian triangulations to the Brownian map.

## Invited Talks (40 min.)

## **Bigeodesics and Polymer Watermelons in Last Passage Percolation**

Riddhipratim Basu  
(Tata Institute of Fundamental Research)

I shall describe two results for the exactly solvable model of exponential last passage percolation that are connected by the common theme of understanding rarity of multiple disjoint geodesics across on-scale (i.e.,  $n \times n^{2/3}$ ) rectangles. The first result shows that almost surely there does not exist any non-trivial bi-infinite geodesic. In the second, we consider the “polymer watermelon” given by the maximal weight collection of  $k$  disjoint geodesics between origin and the point  $(n, n)$  and identify its fluctuation exponents in  $k$ . I shall also discuss the role of integrable probability estimates in these results and some possible extensions if time permits.

## **Branching Particle Systems with Selection and Free Boundary Problems**

Julien Berestycki (The University of Oxford)

The N-Branching Brownian motion is a particle system in which independent particles move on the real-line as Brownian motions, branch at rate one and the total population size is kept constant equal to  $N$  by removing the leftmost particle at each branching event. This model, which was introduced by Brunet and Derrida to study certain noisy reaction-diffusion equations, can also be seen as a model the evolution of a population under selection. Its hydrodynamic limit was shown recently to exist and satisfies a free boundary problem. I will present an analogous result in higher dimension (the so-called Brownian bees model) for which we can prove the existence of the hydrodynamic result, the global existence of the solution to the associated free boundary problem and describe the large time behaviour of the particle system and of its deterministic limit.

This is based on joint works with E. Brunet, J. Nolen and S. Penington.

## **Zero Temperature Limits for Directed Polymers in Random Environment**

Ryoki Fukushima (Kyoto University)

In this talk, I discuss the zero-temperature limit for directed polymer models. In many cases, the limit is simply the first/last passage percolation. However, there are several natural settings where the entropy remains non-degenerate at zero-temperature. I explain a motivation to consider such a situation and present a few results on the continuity of the free energy in the zero-temperature limit.

## Large Deviation for Lozenge Tiling Dynamics

Tadahisa Funaki (Waseda University)

We consider dynamics of lozenge tilings, or equivalently that of dimer covers of honeycomb lattice, or three-dimensional Young diagrams. One can choose the flip rate to satisfy the so-called gradient condition, which was originally proposed by Luby, Randall and Sinclair. The hydrodynamic limit of this model was studied by Laslier and Toninelli (CMP, 2018) under the periodic boundary condition. We discuss the corresponding large deviation principle. This is joint work with Fabio Toninelli.

## Two Manifestations of Rigidity in Point Sets: Forbidden Regions and Maximal Degeneracy

Subhroshekhar Ghosh (National University of Singapore)

A point process is said to be “rigid” if its local observables are completely determined (as deterministic functions of) the point configuration outside a local neighbourhood. For example, it has been shown in earlier work that, in the Ginibre ensemble (a.k.a. the 2D Coulomb gas at inverse temperature  $\beta=2$ ), the point configuration outside any bounded domain determines, almost surely, the number of points in the domain.

In this talk, we will explore two recent manifestations of such rigidity phenomena. For the zeros of the planar Gaussian analytic function, we show that outside every large “hole”, there is a “forbidden region” which contains a vanishing density of points. This should be seen in contrast with the corresponding situation for classically understood models (e.g. random matrix ensembles), where no such effects are known to occur.

In the second part of the talk, we will consider “stealthy” hyperuniform systems, which are systems whose structure function (i.e., the Fourier transform of the two-point correlation) vanishes near the origin. We show that such systems exhibit “maximal degeneracy”, namely the points outside a bounded domain determine, almost surely, the entire point configuration inside the domain. En route, we establish a conjecture of Zhang, Stillinger and Torquato that such systems have (deterministically) bounded holes.

Based on joint works with Joel Lebowitz and Alon Nishry.

## Cardy Embedding of Random Planar Maps

Nina Holden (ETH-ITS Zurich)

A random planar map is a canonical model for a discrete random surface which is studied in probability theory, combinatorics, mathematical physics, and geometry. Liouville quantum gravity is a canonical model for a random 2D Riemannian manifold with roots in the physics literature. In a joint work with Xin Sun, we prove a strong relationship between these two natural models for random surfaces. Namely, we prove that the random planar map converges in the scaling limit to Liouville quantum gravity under a discrete conformal embedding which we call the Cardy embedding. We also prove that the percolation interface on the triangulation converges to the Schramm-Loewner evolution (SLE) with parameter 6 in a quenched sense.

## Determinantal Structures in the $q$ -Whittaker Measure

Takashi Imamura (Chiba University)

The  $q$ -Whittaker measures are probability measures on partition and are written in terms of a product of two  $q$ -Whittaker functions. Since the introduction by Borodin and Corwin in 2011, it has been playing an important role in recent progresses in the integrable probability. Their interesting feature is that the  $q$ -Laplace transform of some marginal distributions can be represented as a single Fredholm determinant, from which we can analyze asymptotic behavior of interacting particle processes belonging to the KPZ class. The Fredholm determinant formula was first obtained by Borodin-Corwin by using Macdonald difference operator. Since then, various techniques such as Markov duality, the Yang-Baxter equation etc, have been developed to obtain the determinantal formulas for more generalized setting. All these techniques focus on some mathematical structures of  $q$ -moments of some observables.

In this talk, I will talk on another approach to analyze the  $q$ -Whittaker measures without relying on the  $q$ -moments. We utilize the Ramanujan summation formula and an elliptic version of the Cauchy determinant identity. We also discuss some relations of the  $q$ -Whittaker measures to determinantal point processes. This is the joint work with Matteo Mucciconi and Tomohiro Sasamoto.

# The Laplacian on Some Self-Conformal Fractals and Weyl's Asymptotics for Its Eigenvalues

Naotaka Kajino (Kobe University)

This talk will present the speaker's recent results on the construction of a "geometrically canonical" Laplacian on round Sierpiński carpets invariant with respect to certain Kleinian groups (i.e., discrete groups of Möbius transformations on  $\widehat{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$ ) and on Weyl's asymptotics for its eigenvalues. Here a *round Sierpiński carpet* refers to a subset of  $\widehat{\mathbb{C}}$  homeomorphic to the standard Sierpiński carpet, such that its complement in  $\widehat{\mathbb{C}}$  consists of disjoint open disks in  $\widehat{\mathbb{C}}$ .

The construction of the Laplacian is based on the speaker's preceding study of the simplest case of the *Apollonian gasket*, the compact fractal subset of  $\mathbb{C}$  obtained from an ideal triangle (a triangle formed by mutually tangent three circles) by repeating indefinitely the process of removing the interior of the inner tangent circles of the ideal triangles. On this fractal, Teplyaev (2004) had constructed a canonical Dirichlet form as one with respect to which the coordinate functions on the gasket are harmonic, and the author later proved its uniqueness and discovered an explicit expression of it in terms of the circle packing structure of the gasket.

The expression of the Dirichlet form obtained for the Apollonian gasket in fact makes sense on general circle packing fractals, including round Sierpiński carpets, and defines (a candidate of) a "geometrically canonical" Laplacian on such fractals. When the circle packing fractal is the limit set (i.e., the minimum invariant non-empty compact set) of a certain class of Kleinian groups, some explicit combinatorial structure of the fractal is known and makes it possible to prove Weyl's asymptotic formula for the eigenvalues of this Laplacian, which is of the same form as the circle-counting asymptotic formula by Oh and Shah [Invent. Math. **187** (2012), 1–35].

The overall structure of the proof of Weyl's asymptotic formula is the same as in the case of the Apollonian gasket and is based on a serious application of Kesten's renewal theorem [Ann. Probab. **2** (1974), 355–386] to a certain Markov chain on the "*space of all possible Euclidean shapes*" of the fractal. There is, however, a crucial difficulty in the case of a round Sierpiński carpet; since it is *infinitely ramified*, i.e., the cells in its cellular decomposition intersect on infinite sets, it is highly non-trivial to show that the principal order term of the eigenvalue asymptotics is not affected by the cellular decomposition, namely by assigning the vanishing Dirichlet boundary condition on the boundary of the cells.

If time permits, a possible approach toward extensions to the case of self-conformal fractals in  $\mathbb{C}$  consisting of nowhere rectifiable curves will also be mentioned.



## Gaussian Free Fields with Boundary Points, Multiple SLEs, and Log-Gases

Makoto Katori (Chuo University)

A quantum surface (QS) (resp. an imaginary surface (IS)) is an equivalence class of pairs of simply commenced domains  $D \subsetneq \mathbb{C}$  and the Gaussian free fields (GFFs) on  $D$  with the free (resp. Dirichlet) boundary condition induced by the conformal equivalence for random metric spaces. We define a QS with  $N+1$  marked boundary points (MBPs) and an IS with  $N+1$  boundary condition changing points (BCCPs) on  $\partial D$  with  $N \in \mathbb{Z}_{\geq 1}$ , in which the real (resp. imaginary) part of the sum of  $\mathbb{C}$ -valued logarithmic (2D Coulomb) potentials arising from the MBPs (resp. BCCPs) is added to GFF in  $D$ . We consider the situation such that the boundary points evolve in time as a stochastic log-gas on  $\partial D$  and multiple random slits are generated in  $D$  by the multiple Schramm–Loewner evolution (SLE) driven by that stochastic log-gas. Then we cut the domain  $D$  along the SLE slits, restrict the GFF on the resulting domain, and pull it back to  $D$  following the reverse flow of the multiple SLE. We prove that if the log-gases on  $\partial D$  follow the stochastic differential equations well-studied in random matrix theory (*e.g.*, Dyson’s Brownian motion model, the Bru–Wishart processes), and parameters are properly chosen, then the coupled systems of GFFs and multiple SLE slits provide stationary processes. The obtained random systems are used to solve interesting geometric problems called the conformal welding problem and the flow line problem. The present study extends the previous results for a QS with two MBPs reported by Sheffield and for an IS with two BCCPs by Miller and Sheffield. This is a joint work with Shinji Koshida (Chuo University); see <https://arxiv.org/abs/1903.09925>.

## Approach to the Quantum Field with Exponential Interactions by Singular SPDEs

Seiichiro Kusuoka (Kyoto University)

We consider the stochastic quantization of the quantum field model with exponential interactions on the two-dimensional torus, which is called Hoegh-Krohn model. The model has been studied by quasi-regular Dirichlet forms. In this talk, we study the model by singular stochastic differential equations, which is recently developed. By the method, we construct the time-global solution and the invariant probability measure of the stochastic quantization, and see the relation to the process obtained by quasi-regular Dirichlet forms. This is a joint work with Masato Hoshino and Hiroshi Kawabi.

## A Class of Positive Functions on the Circle and Their Applications on Hankel Operators

Yanqi Qiu (Chinese Academy of Sciences)

We propose a definition of hyper-positive functions on the circle related to stochastic processes on homogeneous trees. After giving a complete criterion of such functions, we are going to talk about their applications on inequalities of Hankel operators. The talk is based on a joint work with Zipeng WANG.

# Isomorphism Theorems and the Sign Cluster Geometry of the Gaussian Free Field

Pierre-François Rodriguez  
(Institut des Hautes Études Scientifiques)

We consider the Gaussian free field (GFF) on a large class of transient weighted graphs  $G$ , and prove that its sign clusters contain an infinite connected component. In fact, we show that the sign clusters fall into a regime of strong supercriticality, in which two infinite sign clusters dominate (one for each sign), and finite sign clusters are necessarily tiny, with overwhelming probability. Examples of graphs  $G$  belonging to this class include cases in which the random walk on  $G$  exhibits anomalous diffusive behavior. Among other things, our proof exploits a certain relation (isomorphism theorem) relating the GFF to Sznitman’s random interacements. Our findings also imply the existence of a nontrivial percolating regime for the vacant set of random interacements on  $G$  and relate to certain disconnection problems for random walks. Based on joint work with A. Prévost and A. Drewitz (U. Köln).

## Critical Two-Point Function for Long-Range Models with Power-Law Couplings: The Marginal Case for $d \geq d_c$

Akira Sakai (Hokkaido University)

Consider the long-range models on  $Z^d$  of random walk, self-avoiding walk, percolation and the Ising model, whose translation-invariant 1-step distribution/coupling coefficient decays as  $|x|^{-d-a}$  for some  $a > 0$ . In the previous work (Ann. Probab., 43, 639–681, 2015), we have shown in a unified fashion for all  $a$  other than 2 that, assuming a bound on the “derivative” of the  $n$ -step distribution (the compound-zeta distribution satisfies this assumed bound), the critical two-point function  $G_{p_c}(x)$  decays as  $|x|^{a \wedge 2 - d}$  above the upper-critical dimension  $d_c = (a \wedge 2)m$ , where  $m = 2$  for self-avoiding walk and the Ising model and  $m = 3$  for percolation.

In this talk, I will show in a much simpler way, without assuming a bound on the derivative of the  $n$ -step distribution, that  $G_{p_c}(x)$  for the marginal case  $a = 2$  decays as  $|x|^{2-d}/\log|x|$  whenever  $d \geq d_c$  (with a large spread-out parameter  $L$ ). This solves the conjecture in the previous work, extended all the way down to  $d = d_c$ , and confirms a part of predictions in physics (Brezin, Parisi, Ricci-Tersenghi, J. Stat. Phys., 157, 855–868, 2014). The proof is based on the lace expansion and new convolution bounds on power functions with log corrections.

# Generalized Pitman's Transform and Discrete Integrable Systems

Makiko Sasada (The University of Tokyo)

The Korteweg-de Vries equation (KdV equation) and the Toda lattice are typical and well-known classical integrable systems. For the KdV equation, the (almost-sure) well-posedness of a solution starting from a general ergodic random field on  $\mathbb{R}$ , such as the white noise, is still an open problem, though the invariance of the white noise is known on  $\mathbb{T}$ . On the other hand, for the infinite Toda lattice, the invariance under the generalized Gibbs ensembles (GGE) are standard. Recently, Spohn shows that the generalized Gibbs free energy of the Toda chain is related to the  $\beta$ -ensembles of random matrix theory in the mean-field regime and obtained an exact variational formula for the density of states of the Lax matrix, when its matrix elements are distributed according to some GGE.

In this talk, I will present our recent results on discrete time versions of the KdV equation and the Toda lattice. First, I will introduce some generalization of Pitman's transform and show that the dynamics of several discrete integrable systems, such as the discrete KdV equation, the ultra-discrete KdV equation, the discrete Toda lattice and the ultra-discrete Toda lattice are given by them. We apply this observation to define the dynamics uniquely on the infinite product space and study their invariant measures. If time allows, I will also talk about the generalized Gibbs free energy of the discrete Toda chain and its connection to the Wishart  $\beta$ -ensembles of random matrix theory.

This talk is mostly based on a joint work with David Croydon, Tsuyoshi Kato and Satoshi Tsujimoto.

## The Cut-Off Phenomenon for the Random-Cluster Model

Insuk Seo (Seoul National University)

In this presentation, we consider the random-cluster model which is a generalization of the standard edge percolation model. For the random-cluster model on lattice, we prove that the Glauber dynamics exhibits a phenomenon known as the cut-off, especially for the very subcritical regime for all dimensions. This is a joint work with Shirshendu Ganguly.

## Scaling Limit of Uniform Spanning Tree in Three Dimensions

Daisuke Shiraishi (Kyoto University)

We will show that the properly rescaled three-dimensional uniform spanning tree converges weakly with respect to a Gromov-Hausdorff-Prohorov-type topology in a space whose elements are measured, rooted real trees continuously embedded into Euclidean space. We will describe various properties of the intrinsic metrics, measures and embeddings of the limit in this space. This is based on a joint work with Omer Angel (UBC), David Croydon (Kyoto University) and Sarai Hernandez Torres (UBC).

## Lower-Tail Large Deviations of the KPZ Equation

Li-Cheng Tsai (Rutgers University)

Consider the solution of the KPZ equation with the narrow wedge initial condition. We prove the one-point, lower-tail Large Deviation Principle (LDP) of the solution, with time  $t \rightarrow \infty$  being the scaling parameter, and with an explicit rate function. This result confirms existing physics predictions. We utilize a formula from Borodin and Gorin (2016) to convert the LDP of the KPZ equation to calculating an exponential moment of the Airy point process, and analyze the latter via stochastic Airy operator and Riccati transform.

## Stochastic Cahn-Hilliard Equation

Rongchan Zhu (Beijing Institute of Technology)

We study the asymptotic limit, as  $\varepsilon \searrow 0$ , of solutions to the stochastic Cahn-Hilliard equation:

$$\partial_t u^\varepsilon = \Delta \left( -\varepsilon \Delta u^\varepsilon + \frac{1}{\varepsilon} f(u^\varepsilon) \right) + \dot{\mathcal{W}}_t^\varepsilon,$$

where  $\mathcal{W}^\varepsilon = \varepsilon^\sigma W$  or  $\mathcal{W}^\varepsilon = \varepsilon^\sigma W^\varepsilon$ ,  $W$  is a  $Q$ -Wiener process and  $W^\varepsilon$  is smooth in time and converges to  $W$  as  $\varepsilon \searrow 0$ . In the case that  $\mathcal{W}^\varepsilon = \varepsilon^\sigma W$ , we prove that for all  $\sigma > \frac{1}{2}$ , the solution  $u^\varepsilon$  converges to a weak solution to an appropriately defined limit of the deterministic Cahn-Hilliard equation. In radial symmetric case we prove that for all  $\sigma \geq \frac{1}{2}$ ,  $u^\varepsilon$  converges to the deterministic Hele-Shaw model. In the case that  $\mathcal{W}^\varepsilon = \varepsilon^\sigma W^\varepsilon$ , we prove that for all  $\sigma > 0$ ,  $u^\varepsilon$  converges to the weak solution to the deterministic limit Cahn-Hilliard equation. In radial symmetric case we prove that  $u^\varepsilon$  converges to deterministic Hele-Shaw model when  $\sigma > 0$  and converges to a stochastic model related to stochastic Hele-Shaw model when  $\sigma = 0$ .

## Invited Talks (20 min.)

## **Exceptional Points of Two-Dimensional Random Walks at Multiples of the Cover Time**

Yoshihiro Abe (Chiba University)

I will talk about the local time of the continuous-time simple random walk on a large finite subset of the two-dimensional lattice with the wired boundary condition. The running time of the walk is parametrized by the local time at the “boundary vertex” and approximated by a multiple of the cover time. The talk will focus on statistics of avoided points (those not visited at all). This is based on joint work with Marek Biskup (UCLA).

## **Paracontrolled Calculus and Regularity Structures**

Masato Hoshino (Kyushu University)

First we introduce the Bailleul-Hoshino’s result, which links the theory of regularity structures with the paracontrolled calculus. As an application of their result, we give another algebraic proof of the multicomponent commutator estimate, which is a general version of the Gubinelli-Imkeller-Perkowski’s commutator estimate.

## **Natural Parametrization for the Scaling Limit of Loop-Erased Random Walk in Three Dimensions**

Xinyi Li (The University of Chicago)

In this talk, I will talk about some recent progress on loop-erased random walk (LERW), a model that plays a central role in statistic physics. I will focus on the case of three dimensions and talk about its convergence in natural parametrization to its scaling limit. This is joint work with Daisuke Shiraishi (Kyoto).

## **Universality Classes for General Random Matrix Flows**

Jacek Małecki  
(Wrocław University of Science and Technology)

We consider matrix-valued processes described as solutions to stochastic differential equations of very general form. We study the family of the empirical measure-valued processes constructed for the corresponding eigenvalues. We show that the family indexed by the size of the matrix is tight under very mild assumptions on the coefficients of the initial SDE. We characterize the limiting distributions of its sub-sequences as solutions to the integro-differential equation. We use the result to study universality classes of random matrix flows. These generalize the classical results related to Dyson Brownian motion and squared Bessel particle systems, but even in these cases we show new phenomenons as the lack of uniqueness of solution and existence of the generalized Marchenko-Pastur distributions supported on the real line. Assuming the uniqueness of the solution to the integro-differential equation we characterize the limiting process as a free diffusion solving free SDE. This is based on a joint work with José Luis Pérez.

## **Divergence of Non-Random Fluctuation in First-Passage Percolation**

Shuta Nakajima (Nagoya University)

We consider the First-passage percolation on  $Z^d$ . There are two types of fluctuations in First-passage percolation: the random fluctuation and the non-random fluctuation. They are both important and well studied. Although there has been a significant progress of the upper bound due to the development of concentration inequalities, it is hard to get a non-trivial lower bound because of the lack of techniques. In this talk, we discuss the divergence of the non-random fluctuation.

## **Exponents for High Points of Simple Random Walks in Two Dimensions**

Izumi Okada (Kyushu University)

We consider the favorite points of random walk, i.e. points where the local time is a fraction of the maximum. These are conjectured to coincide with the exponents for the numbers of pairs of late points and the one for high points of the Gaussian free field for which their exact values are known. We estimate the exponents for the numbers of a multipoint set of late points, favorite points and high points in average and in probability and verified that the conjecture is correct. In addition, we extend this result to the general domain which a simple random walk frequently visits.

## **Derivation of Viscous Burgers Equations from Weakly Asymmetric Exclusion Processes**

Kenkichi Tsunoda (Osaka University)

We consider in this talk the so-called incompressible limit for the weakly asymmetric exclusion processes. A fundamental question in mathematical physics is the derivation of the master equation of fluids such as the Burgers equation or the Navier-Stokes equation. By the celebrated Varadhan's nongradient method, Esposito, Marra and Yau (1996) and Quastel and Yau (1998) derived the Navier-Stokes equation as the incompressible limits in dimensions strictly larger than 2. On the other hand, the derivation of the Navier-Stokes equation in low dimensions is achieved only from a stochastic lattice gas dynamics with long jumps. In this talk, we discuss the incompressible limit for the weakly asymmetric exclusion processes via recent notable entropy estimates developed by Menezes and Jara. This talk is based on joint work with Milton Jara (IMPA) and Claudio Landim (IMPA).

# **List of Contributed Posters**

(as of July 25)



- Eleanor Archer (The University of Warwick)  
“Brownian Motion on Stable Looptrees”
- Sung-Soo Byun (Seoul National University)  
“Annulus SLE Partition Functions and Martingale-Observables”
- Taiki Endo (Chuo University)  
“Three-Parametric Marcenko–Pastur Density”
- Xiang Fang (National Central University)  
“Random Weighted Shifts”
- Yueyuan Gao (AIST, Tohoku University)  
“Uniqueness of the Entropy Solution of a Stochastic Conservation Law with a  $Q$ -Brownian Motion”
- Masayuki Kageyama (Nagoya City University)  
“A New Criteria of Risk in Markov Decision Processes”
- Yoshinori Kamijima (Hokkaido University)  
“Mean-Field Behavior for the Quantum Ising Model”
- Shinji Koshida (Chuo University)  
“Free Field Approach to the Macdonald Process”
- Atef Lechiheb (The University of Tunis El Manar)  
TBA
- Matteo Mucciconi (Tokyo Institute of Technology)  
“Yang Baxter Field and Stochastic Vertex Models”
- Takuya Murayama (Kyoto University)  
“Loewner Chains and Evolution Families on Parallel Slit Half-Planes”
- Takuya Nakagawa (Ritsumeikan University)  
“ $L^{\alpha-1}$  Distance between Two One-Dimensional Stochastic Differential Equations Driven by a Symmetric  $\alpha$ -Stable Process”
- Shota Osada (Kyushu University)  
“Isomorphisms between DPPs and PPPs”
- Yuki Tokushige (Kyoto University)  
“The Differentiability of the Speed of Biased RWs on Galton-Watson Trees”
- Linjie Zhao (Peking University)  
“Scaling Limits for the Exclusion Process with a Slow Site”