# Stochastic Cahn-Hilliard Equation

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We study the asymptotic limit, as $\varepsilon \searrow 0$, of solutions to the stochastic Cahn-Hilliard equation:

$$\partial_t u^\varepsilon = \Delta \left( -\varepsilon \Delta u^\varepsilon + \frac{1}{\varepsilon} f(u^\varepsilon) \right) + \dot{\mathcal{W}}_t^\varepsilon,$$

where $\mathcal{W}^\varepsilon = \varepsilon^\sigma W$ or $\mathcal{W}^\varepsilon = \varepsilon^\sigma W^\varepsilon$, $W$ is a $Q$-Wiener process and $W^\varepsilon$ is smooth in time and converges to $W$ as $\varepsilon \searrow 0$. In the case that $\mathcal{W}^\varepsilon = \varepsilon^\sigma W$, we prove that for all $\sigma > \frac{1}{2}$, the solution $u^\varepsilon$ converges to a weak solution to an appropriately defined limit of the deterministic Cahn-Hilliard equation. In radial symmetric case we prove that for all $\sigma \geq \frac{1}{2}$, $u^\varepsilon$ converges to the deterministic Hele-Shaw model. In the case that $\mathcal{W}^\varepsilon = \varepsilon^\sigma W^\varepsilon$, we prove that for all $\sigma > 0$, $u^\varepsilon$ converges to the weak solution to the deterministic limit Cahn-Hilliard equation. In radial symmetric case we prove that $u^\varepsilon$ converges to deterministic Hele-Shaw model when $\sigma > 0$ and converges to a stochastic model related to stochastic Hele-Shaw model when $\sigma = 0$.