Generalized Pitman's Transform and Discrete Integrable Systems

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The Korteweg-de Vries equation (KdV equation) and the Toda lattice are typical and well-known classical integrable systems. For the KdV equation, the (almost-sure) well-posedness of a solution starting from a general ergodic random field on $\mathbb{R}$, such as the white noise, is still an open problem, though the invariance of the white noise is known on $\mathbb{T}$. On the other hand, for the infinite Toda lattice, the invariance under the generalized Gibbs ensembles (GGE) are standard. Recently, Spohn shows that the generalized Gibbs free energy of the Toda chain is related to the β-ensembles of random matrix theory in the mean-field regime and obtained an exact variational formula for the density of states of the Lax matrix, when its matrix elements are distributed according to some GGE.

In this talk, I will present our recent results on discrete time versions of the KdV equation and the Toda lattice. First, I will introduce some generalization of Pitman's transform and show that the dynamics of several discrete integrable systems, such as the discrete KdV equation, the ultra-discrete KdV equation, the discrete Toda lattice and the ultra-discrete Toda lattice are given by them. We apply this observation to define the dynamics uniquely on the infinite product space and study their invariant measures. If time allows, I will also talk about the generalized Gibbs free energy of the discrete Toda chain and its connection to the Wishart β-ensembles of random matrix theory.

This talk is mostly based on a joint work with David Croydon, Tsuyoshi Kato and Satoshi Tsujimoto.