Critical Two-Point Function for Long-Range Models with Power-Law Couplings: The Marginal Case for $d \geq d_c$

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Consider the long-range models on Z^d of random walk, self-avoiding walk, percolation and the Ising model, whose translation-invariant 1-step distribution/coupling coefficient decays as $|x|^{-d-a}$ for some $a > 0$. In the previous work (Ann. Probab., 43, 639–681, 2015), we have shown in a unified fashion for all a other than 2 that, assuming a bound on the "derivative" of the n-step distribution (the compound-zeta distribution satisfies this assumed bound), the critical two-point function $G_{p_c}(x)$ decays as $|x|^{a\wedge 2-d}$ above the upper-critical dimension $d_c = (a \wedge 2)m$, where $m = 2$ for self-avoiding walk and the Ising model and $m = 3$ for percolation.

In this talk, I will show in a much simpler way, without assuming a bound on the derivative of the n-step distribution, that $G_{p_c}(x)$ for the marginal case $a = 2$ decays as $|x|^{2-d}/\log|x|$ whenever $d \geq d_c$ (with a large spread-out parameter L). This solves the conjecture in the previous work, extended all the way down to $d = d_c$, and confirms a part of predictions in physics (Brezin, Parisi, Ricci-Tersenghi, J. Stat. Phys., 157, 855–868, 2014). The proof is based on the lace expansion and new convolution bounds on power functions with log corrections.