Gaussian Free Fields with Boundary Points, Multiple SLEs, and Log-Gases

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A quantum surface (QS) (resp. an imaginary surface (IS)) is an equivalence class of pairs of simply commenced domains $D \subsetneq \mathbb{C}$ and the Gaussian free fields (GFFs) on D with the free (resp. Dirichlet) boundary condition induced by the conformal equivalence for random metric spaces. We define a QS with $N+1$ marked boundary points (MBPs) and an IS with $N+1$ boundary condition changing points (BCCPs) on ∂D with $N \in \mathbb{Z}_{\geq 1}$, in which the real (resp. imaginary) part of the sum of $\mathbb{C}$-valued logarithmic (2D Coulomb) potentials arising from the MBPs (resp. BCCPs) is added to GFF in D. We consider the situation such that the boundary points evolve in time as a stochastic log-gas on ∂D and multiple random slits are generated in D by the multiple Schramm–Loewner evolution (SLE) driven by that stochastic log-gas. Then we cut the domain D along the SLE slits, restrict the GFF on the resulting domain, and pull it back to D following the reverse flow of the multiple SLE. We prove that if the log-gases on ∂D follow the stochastic differential equations well-studied in random matrix theory (*e.g.*, Dyson's Brownian motion model, the Bru–Wishart processes), and parameters are properly chosen, then the coupled systems of GFFs and multiple SLE slits provide stationary processes. The obtained random systems are used to solve interesting geometric problems called the conformal welding problem and the flow line problem. The present study extends the previous results for a QS with two MBPs reported by Sheffield and for an IS with two BCCPs by Miller and Sheffield. This is a joint work with Shinji Koshida (Chuo University); see https://arxiv.org/abs/1903.09925.